PhD Core Course on General Relativity

Cornelius Rampf
ICG, University of Portsmouth

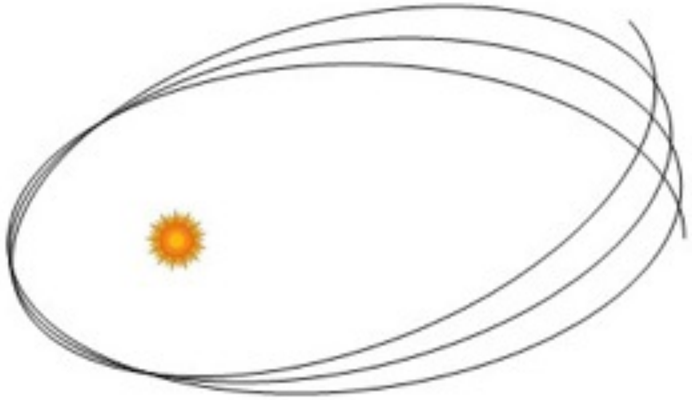
December 8-10-12 2014

GR in a nutshell

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

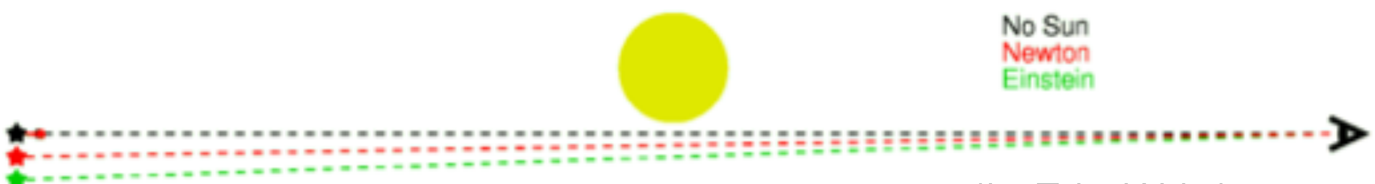
- (Newtonian) action at a distance is just approximation
- information of the gravitational field propagates with speed of light c
- In a freely falling reference frame, Special Relativity must hold (equivalence principle)
- Space-time is a four-dimensional manifold
- Gravity as a geometric property of space-time
  -> gravitational field is described by the Riemann tensor
- Metric coefficients are the dynamical fields

Why GR?

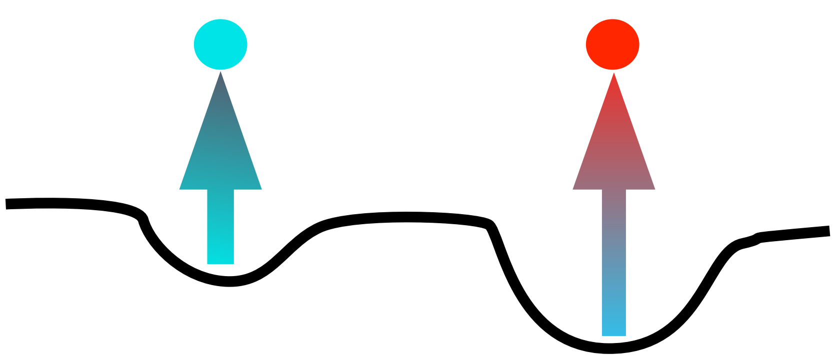


- Introduced in 1915, GR passed several tests:
  + perihelion precision of Mercury's orbit (7%)
  + deflection of light by sun (“50%”)
  + gravitational redshift of light, clocks (100%)

  contribution from GR



credit: E.L. Wright



- Applications in astrophysics, cosmology, GPS,…

ICG Portsmouth

Why (not) GR?

- In general, more accurate description than the Newtonian approximation

- Fully covariant theory, no separation of space-time

- Consists a wealth of additional information (e.g. gravitational waves, lensing)

- Interpretation is not always straightforward

"Make everything as simple as possible, but not simpler."
Einstein

ICG Portsmouth

Outline of the Course

LECTURE 1

- ☐ Special Relativity and flat space-time
- ☐ Manifolds
- ☐ Curvature & Gravitation
- ☐ Schwarzschild solution, black holes
- ☐ ADM formalism
- ☐ “Weyl" formalism of GR
- ☐ Cosmological perturbations: standard, gradient expansion, …
- ☐ Eulerian/Lagrangian fluid descriptions in GR

ICG Portsmouth

Special Relativity

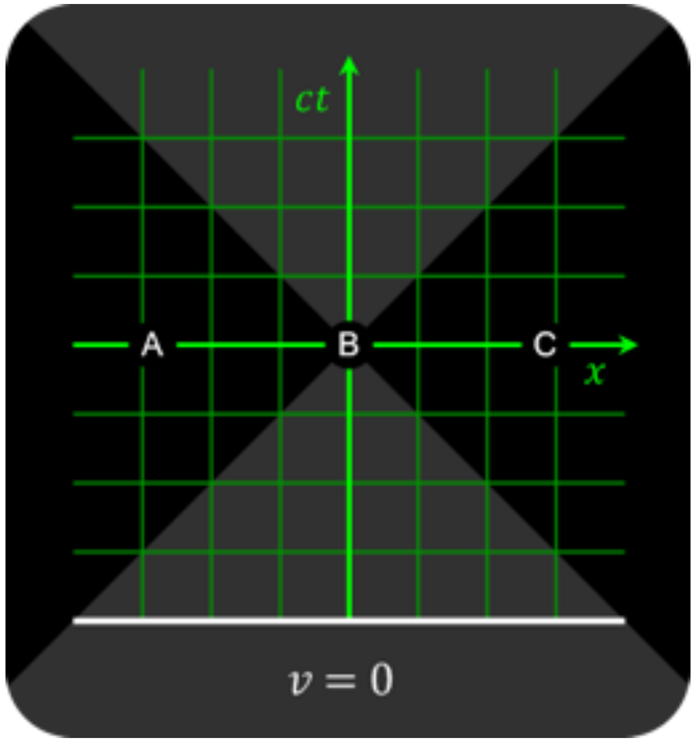
- Speed of light *c* is invariant in all reference frames
- Law of physics are invariant in all inertial frames
- Space and time are linked together
- Minkowski line element: $$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \overset{\text{Einstein's sum convention}}{=} -c\,\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2$$
- coordinates: $$x^\mu = (ct, x^i) \qquad \mu, \nu, \ldots = 0, 1, 2, 3$$
- Minkowski metric $$\eta_{\mu\nu} = \mathrm{Diag}(-1, 1, 1, 1)$$

(Lorentz signature)

SR key assumptions

- Speed of light *c* is invariant in all reference frames
- Law of physics are invariant in all inertial frames

- Relativity of simultaneity
- Time dilation
- Length contraction
- non-additivity of velocities

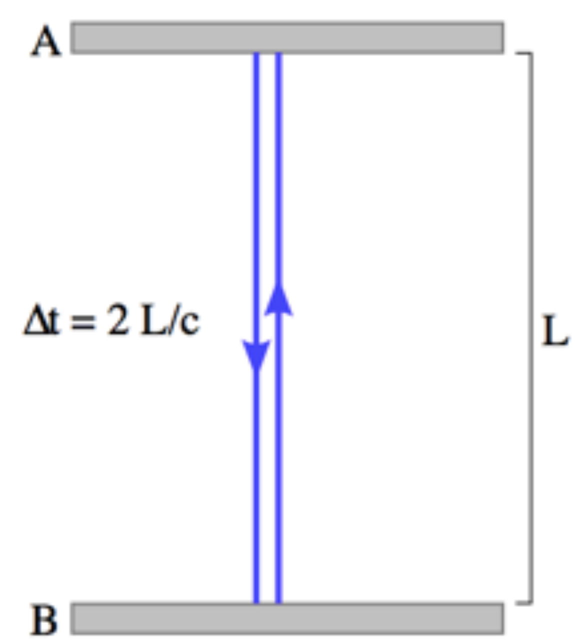


v velocity of observer

SR key assumptions

- Speed of light *c* is invariant in all reference frames
- Law of physics are invariant in all inertial frames

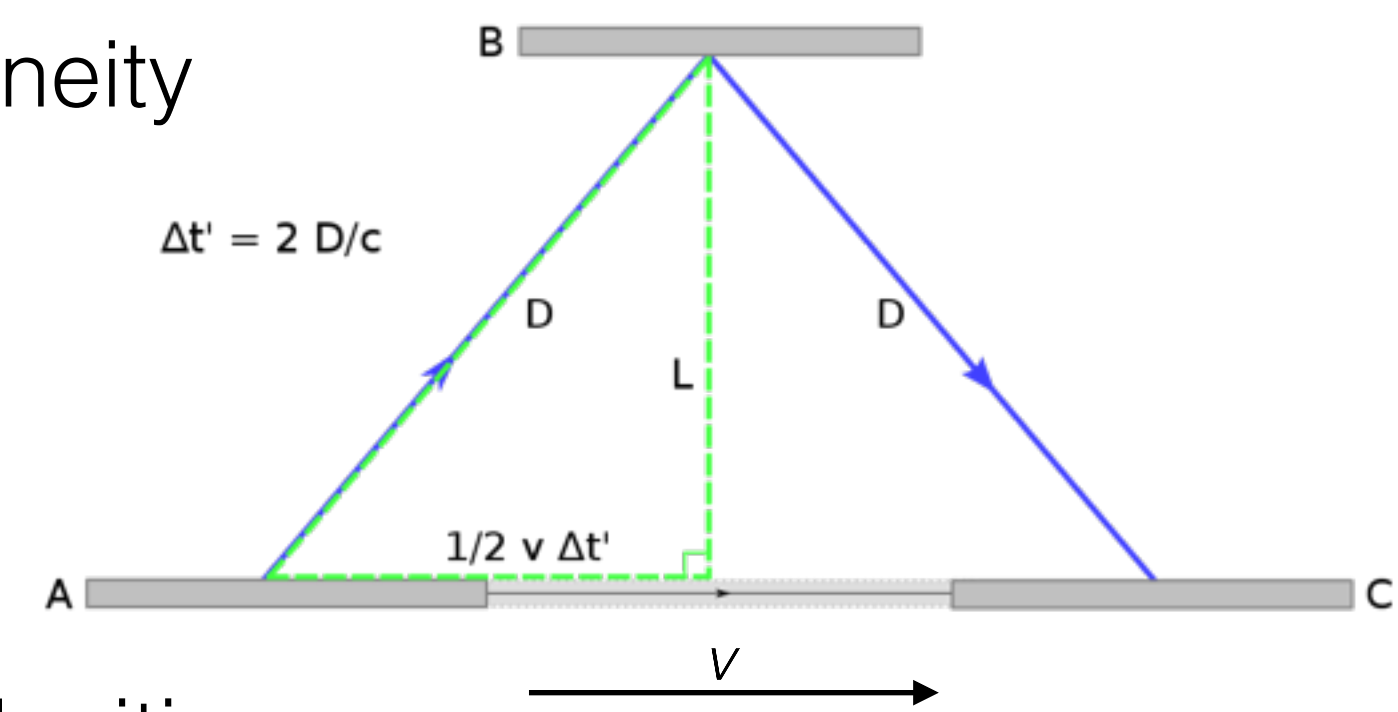
- Relativity of simultaneity
- Time dilation
- Length contraction
- non-additivity of velocities



SR key assumptions

- Speed of light *c* is invariant in all reference frames
- Law of physics are invariant in all inertial frames

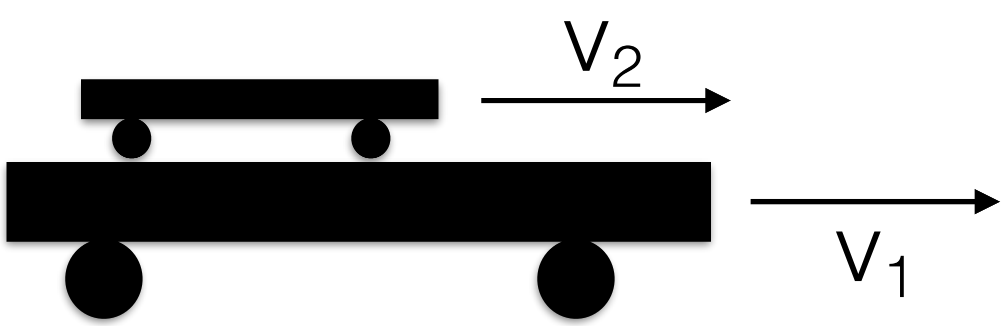
- Relativity of simultaneity
- Time dilation
- Length contraction
- non-additivity of velocities



SR key assumptions

- Speed of light *c* is invariant in all reference frames
- Law of physics are invariant in all inertial frames

- Relativity of simultaneity
- Time dilation
- Length contraction
- non-additivity of velocities



SR III

- $\mathrm{d}s^2$ invariant

- connecting inertial frames: Lorentz transformation

$$\mathrm{d}x^\mu = \left(\frac{\partial x^\mu}{\partial y^\nu}\right)\mathrm{d}y^\nu\,, \quad \left(\frac{\partial x^\mu}{\partial y^\nu}\right)^{-1}\mathrm{d}x^\mu = \mathrm{d}y^\nu, \quad \left(\frac{\partial x^\mu}{\partial y^\nu}\right)\left(\frac{\partial y^\nu}{\partial x^\alpha}\right) = \delta^\mu_\alpha$$

- Linear transformation (rotations, boosts): $x^{\mu'} = \Lambda^{\mu'}{}_{\nu} x^\nu$

Lorentz group

SR IV

- Lorentz boost along *x*-direction: $\gamma = 1/\sqrt{1 - v^2/c^2}$

$$t' = \gamma(t - vx/c^2)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

- Proper time: $c^2 \mathrm{d}\tau^2 \equiv -\mathrm{d}s^2$

- Time-like, space-like, null events

$$\mathrm{d}s^2 < 0\,, \quad \mathrm{d}s^2 > 0\,, \quad \mathrm{d}s^2 = 0$$

causal connection possible

no causal connection

ICG Portsmouth

Manifolds & Curvature

Manifolds & Curvature

- How to obtain a Lorentz-invariant theory of gravity??
  Replace Minkowski $\longrightarrow$ curved space-time

curvature $\underset{\longrightarrow}{\overset{\longleftarrow}{\text{reacts}}}$ **energy & momentum**

- Mathematical framework of curved spaces: **Manifolds**

ICG Portsmouth