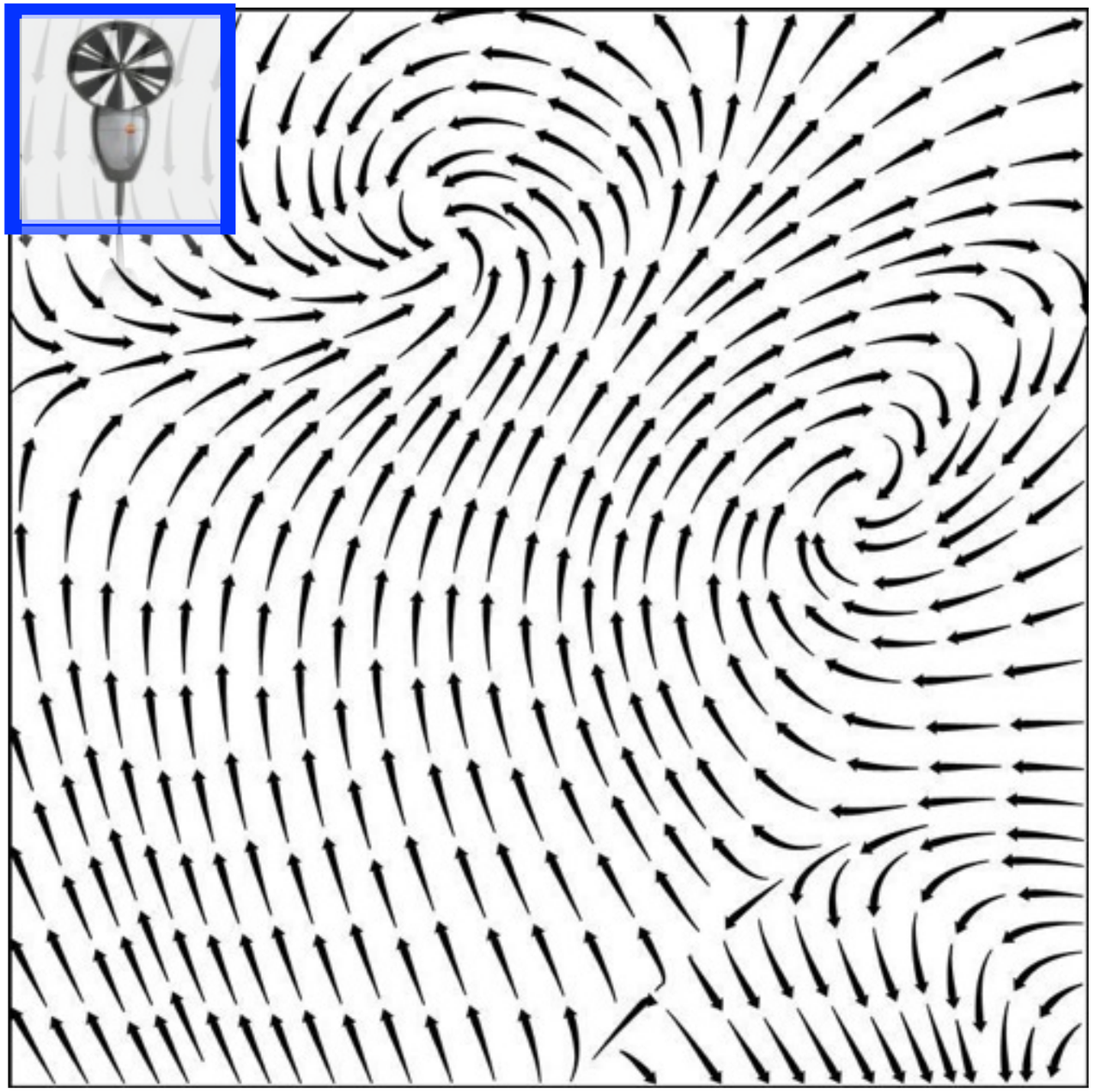
Eulerian & Lagrangian flows in fluid mechanics

(in a nutshell)

Fluid flow: The **Eulerian** way

The Eulerian measurement:

- Fix a **control volume**
- Measure density, velocity field (with a probe)



arrows denote a velocity field

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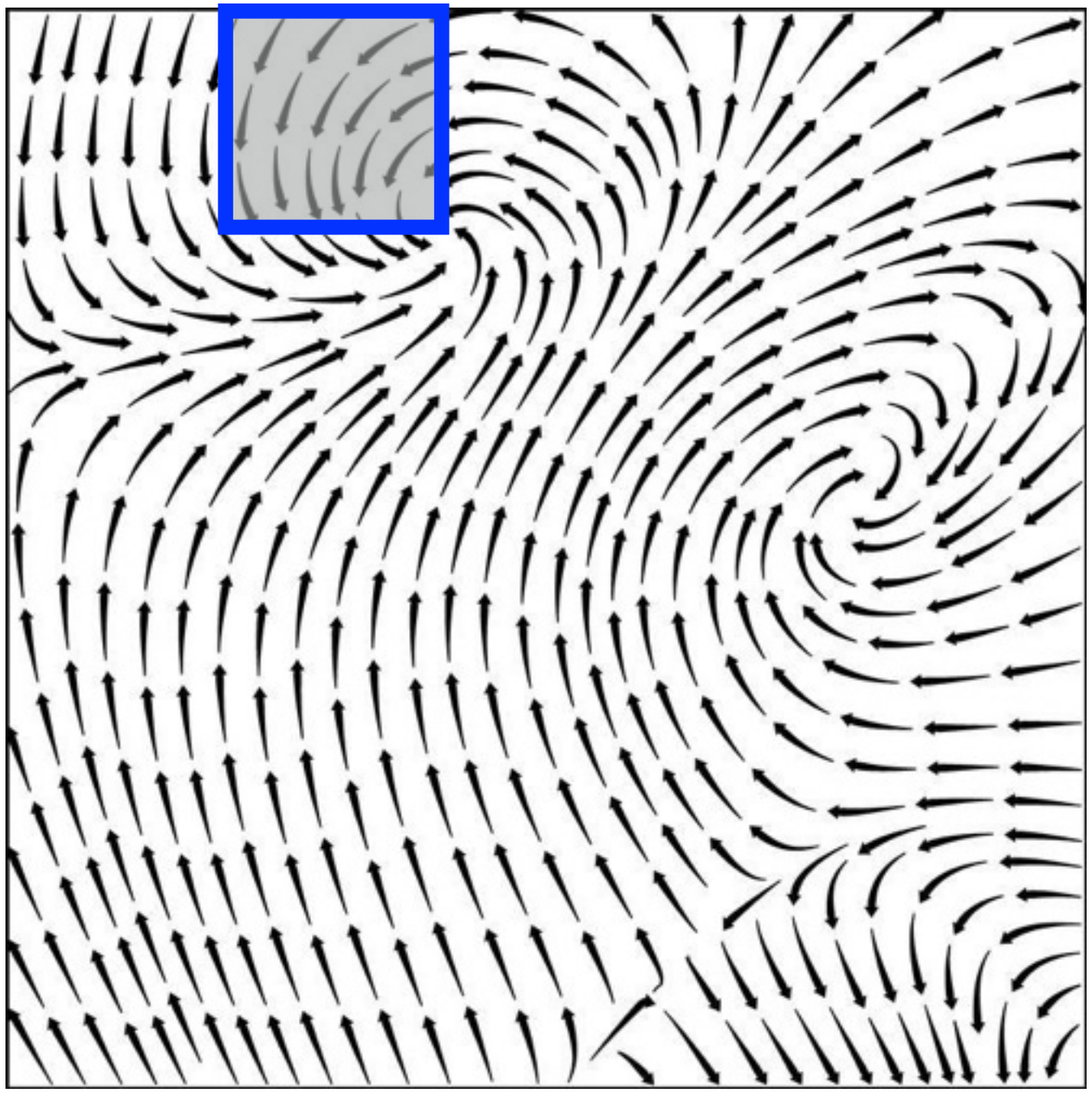

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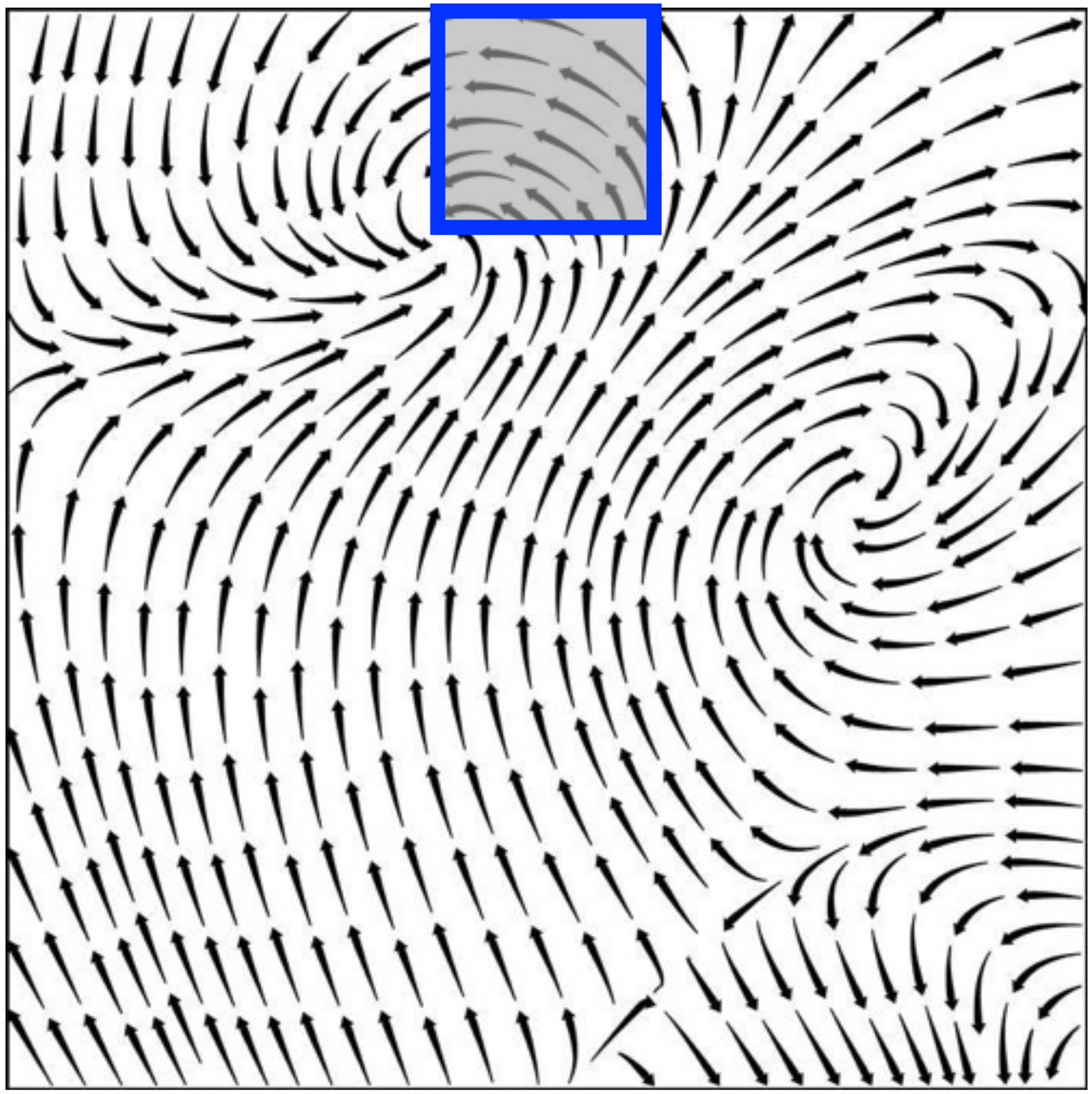

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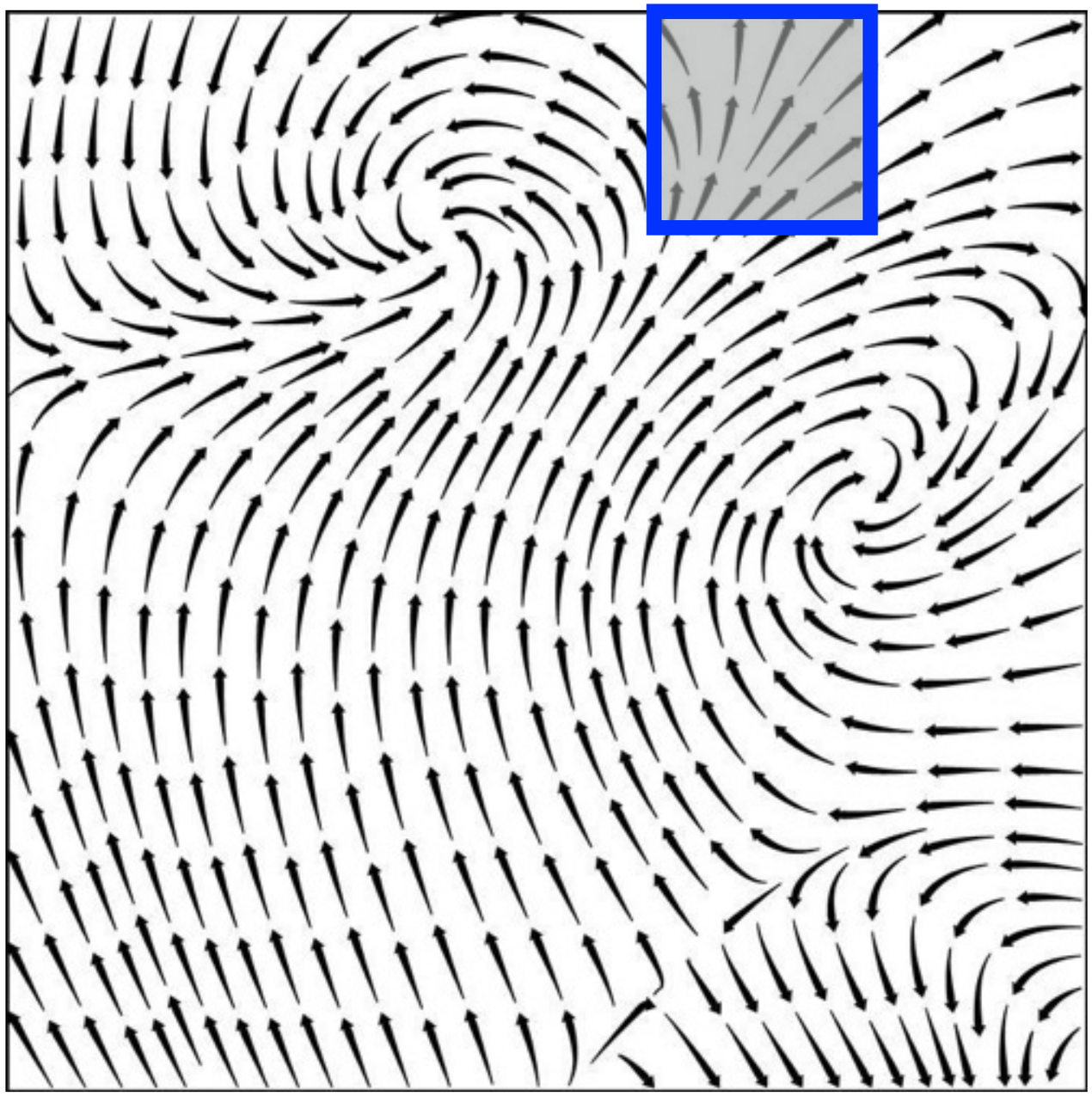

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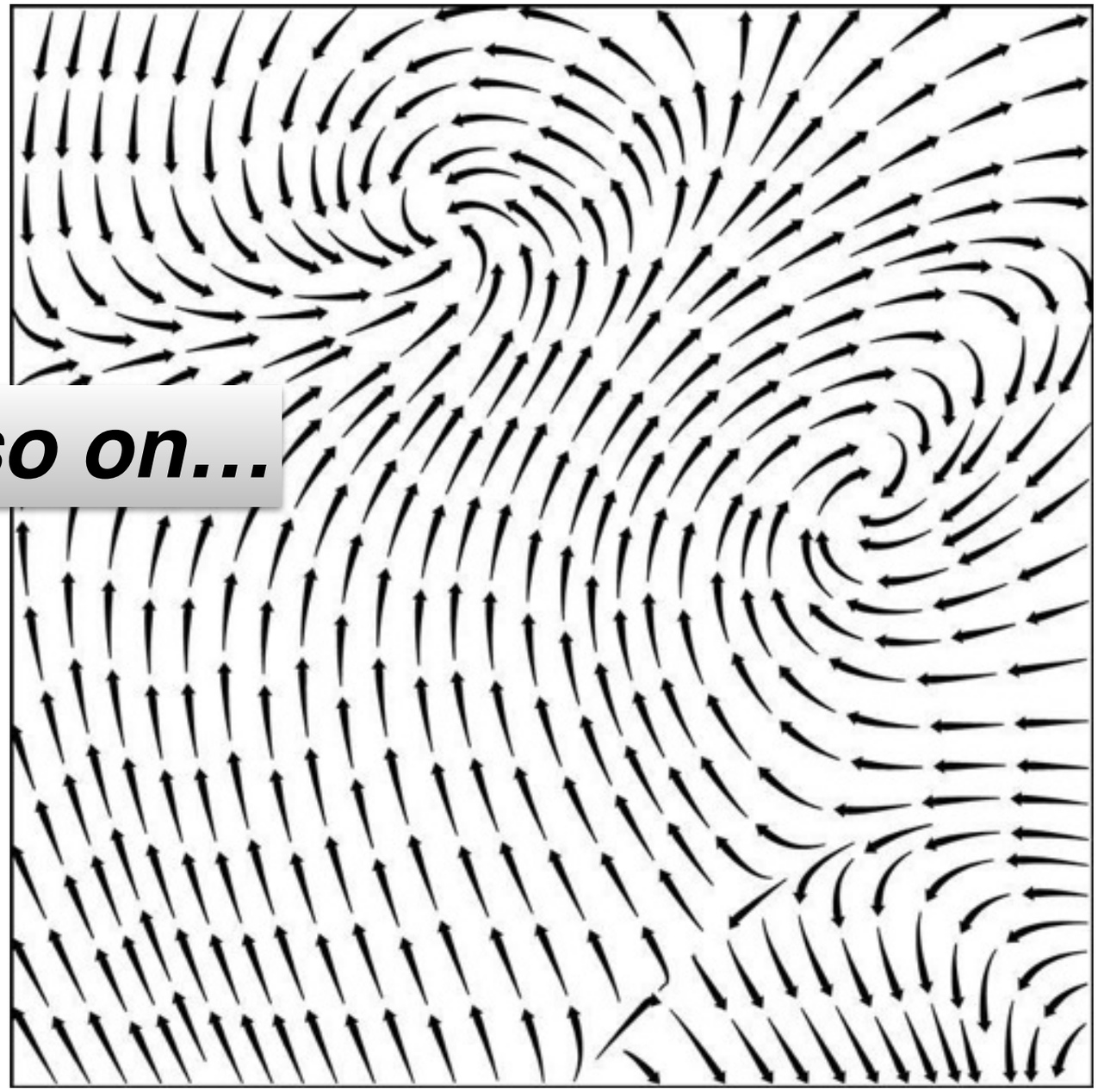
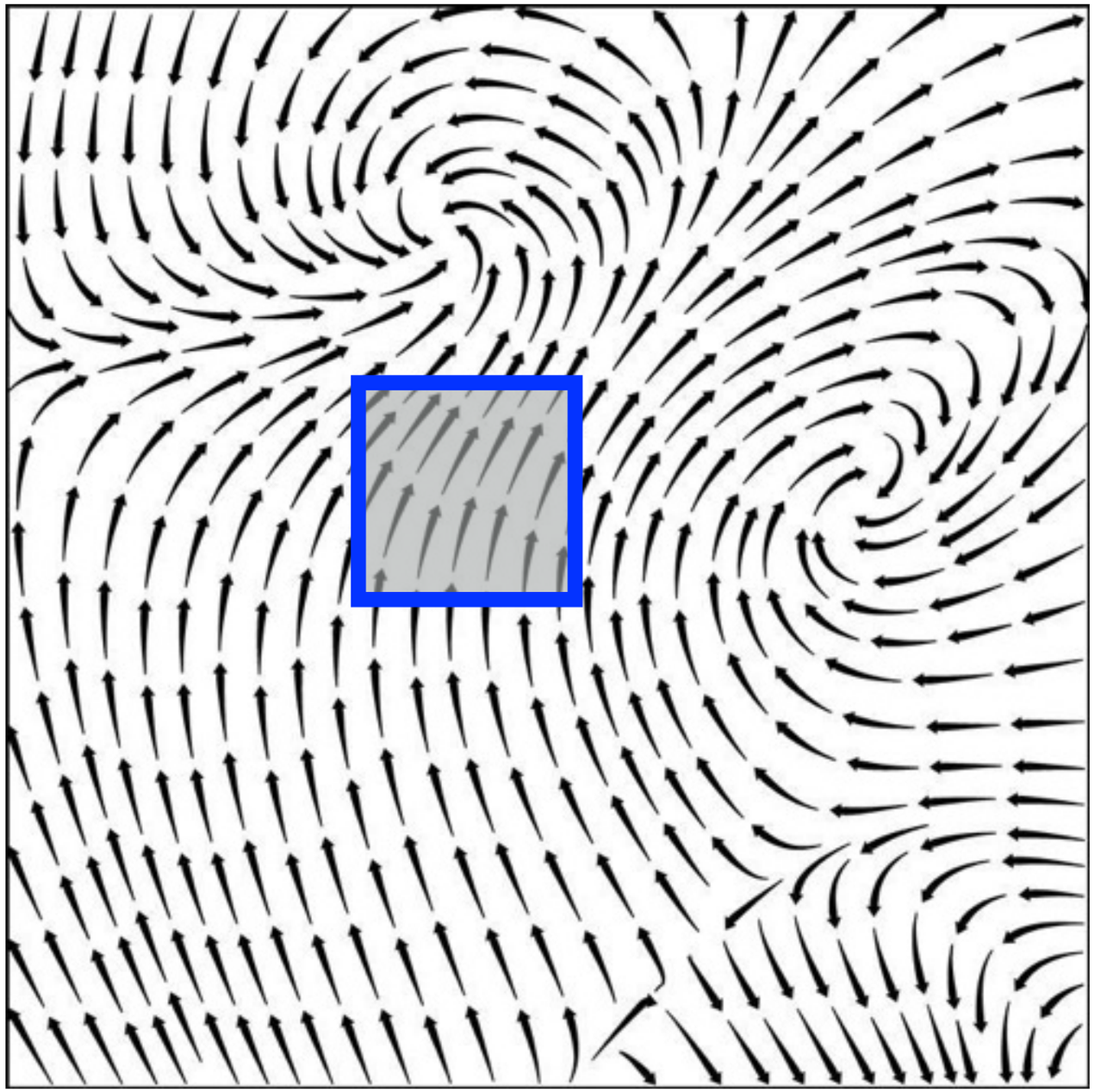
and so on…



Fluid flow: The **Lagrangian** way

Lagrangian measurement:

- Fix a **fluid volume** (agrees with a Eulerian volume at **initial time t_0**)
- **Follow** it during the time evolution



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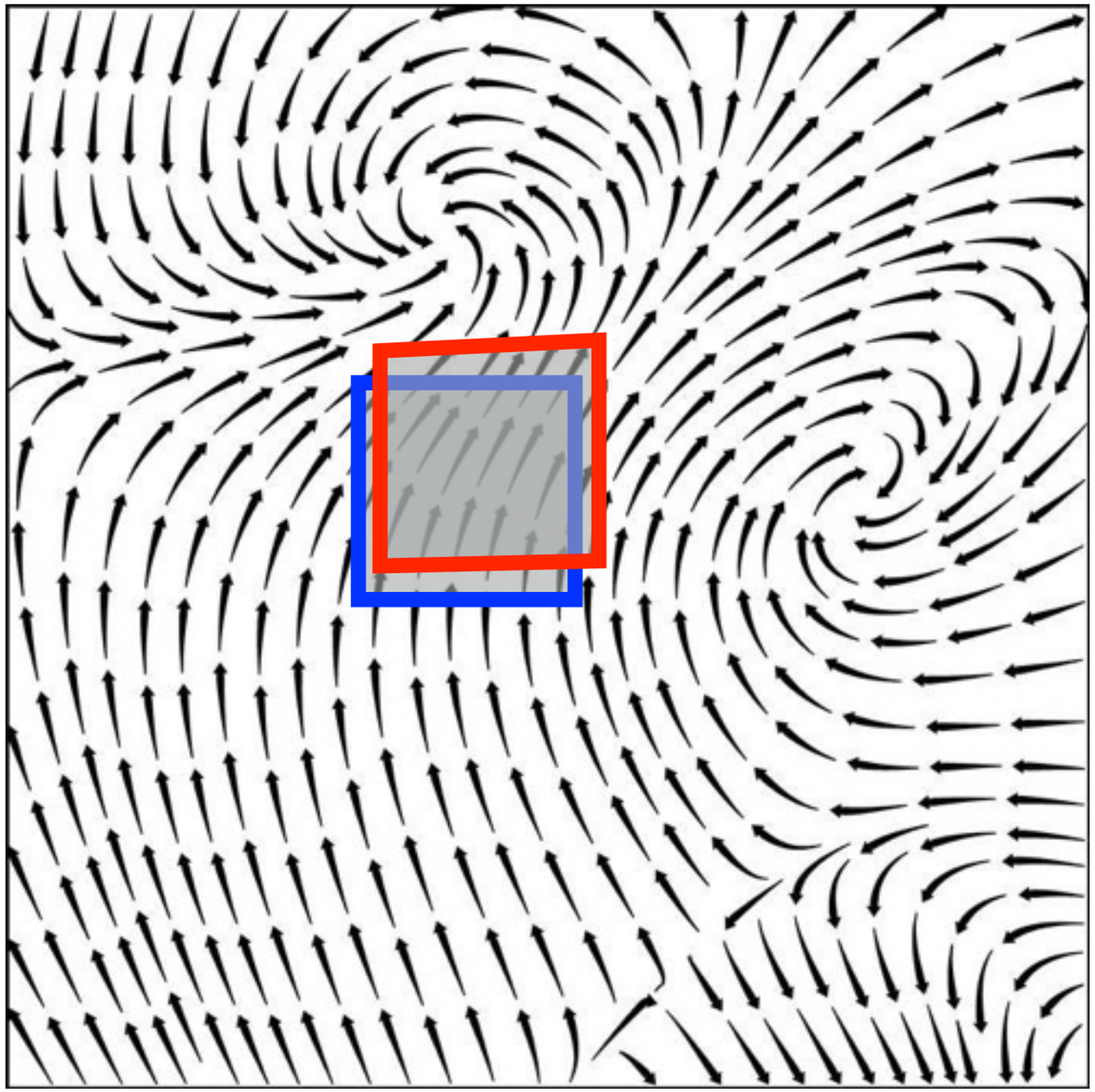

$t_0 > t_1$

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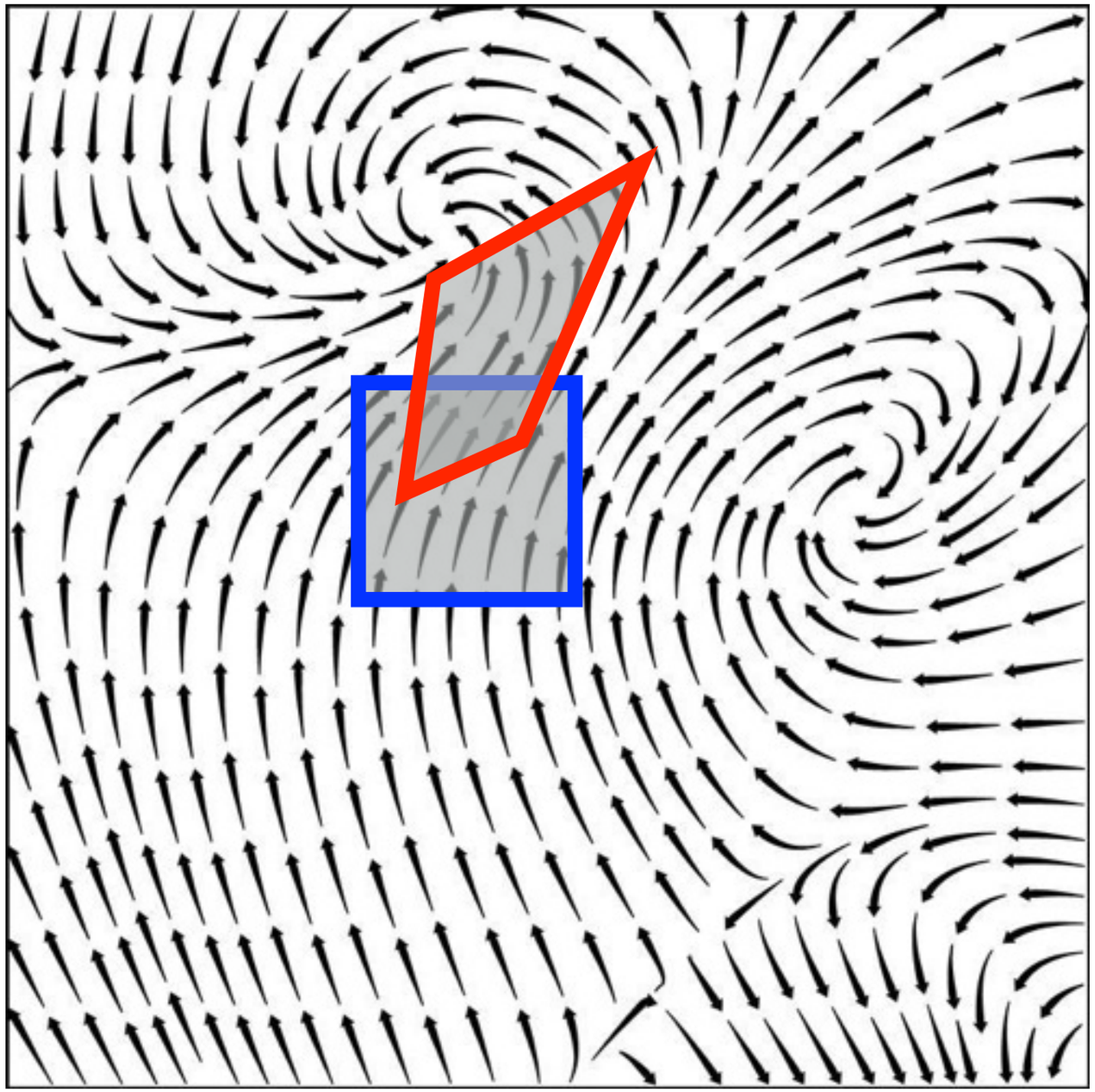

$t_0 > t_1 > t_2$

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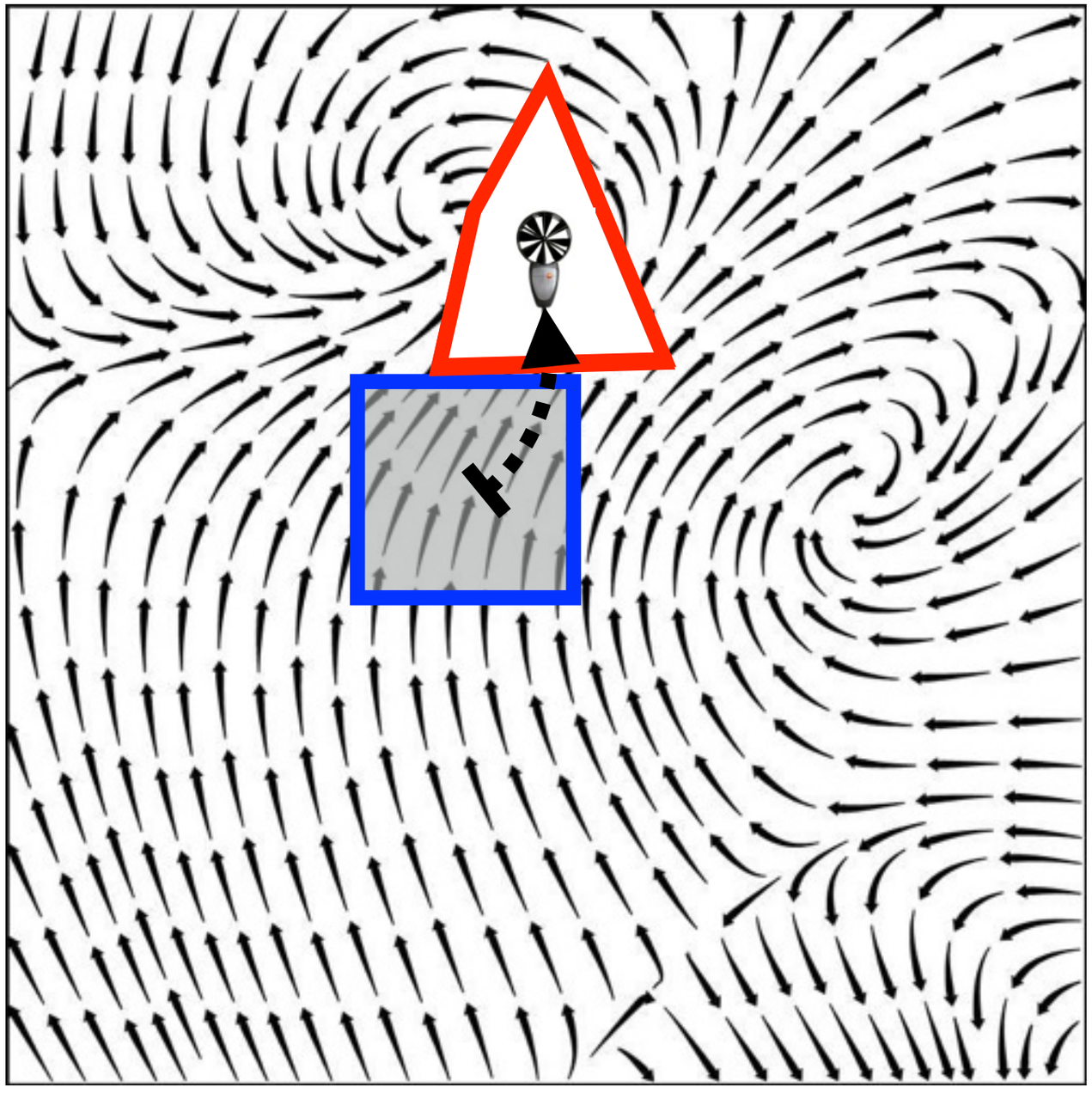

$t_0 > t_1 > t_2 > t_3$

The probe is following the trajectory of the fluid element; measures “something different” compared to the Eulerian probe!

fluid density, velocity, etc. are fundamentally different measures in the **Eulerian** & **Lagrangian** frames!

Observer measures the overall **density** & **velocity fields**

Trajectory of the fluid element contains information about **volume deformation** & **velocity**

$$\rho = \frac{m}{V}$$

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