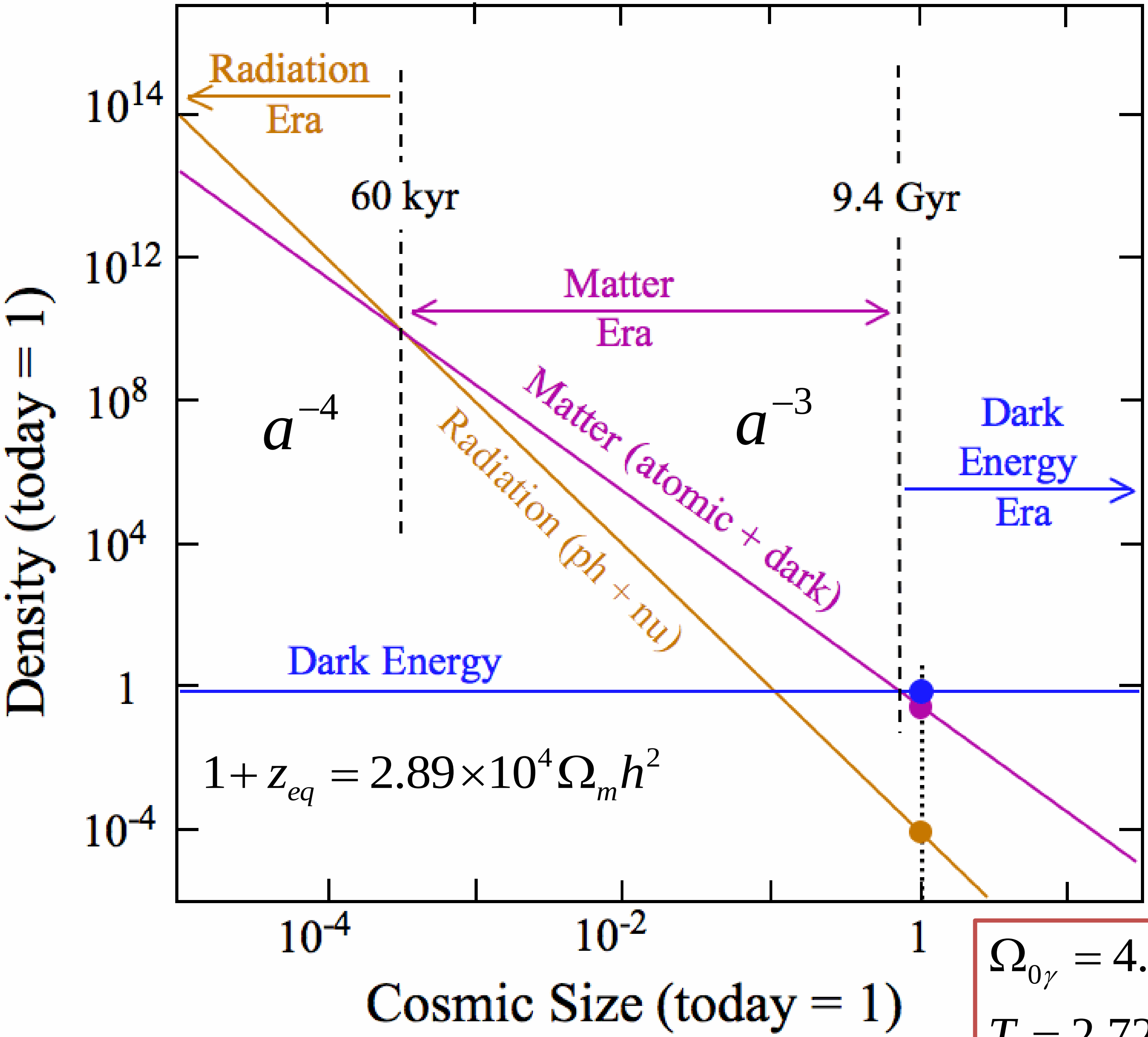
The Early Universe

Thermal history

Nucleosynthesis

The CMB



$$\Omega_{0\gamma}=4.19\times10^{-5}h^{-2}$$

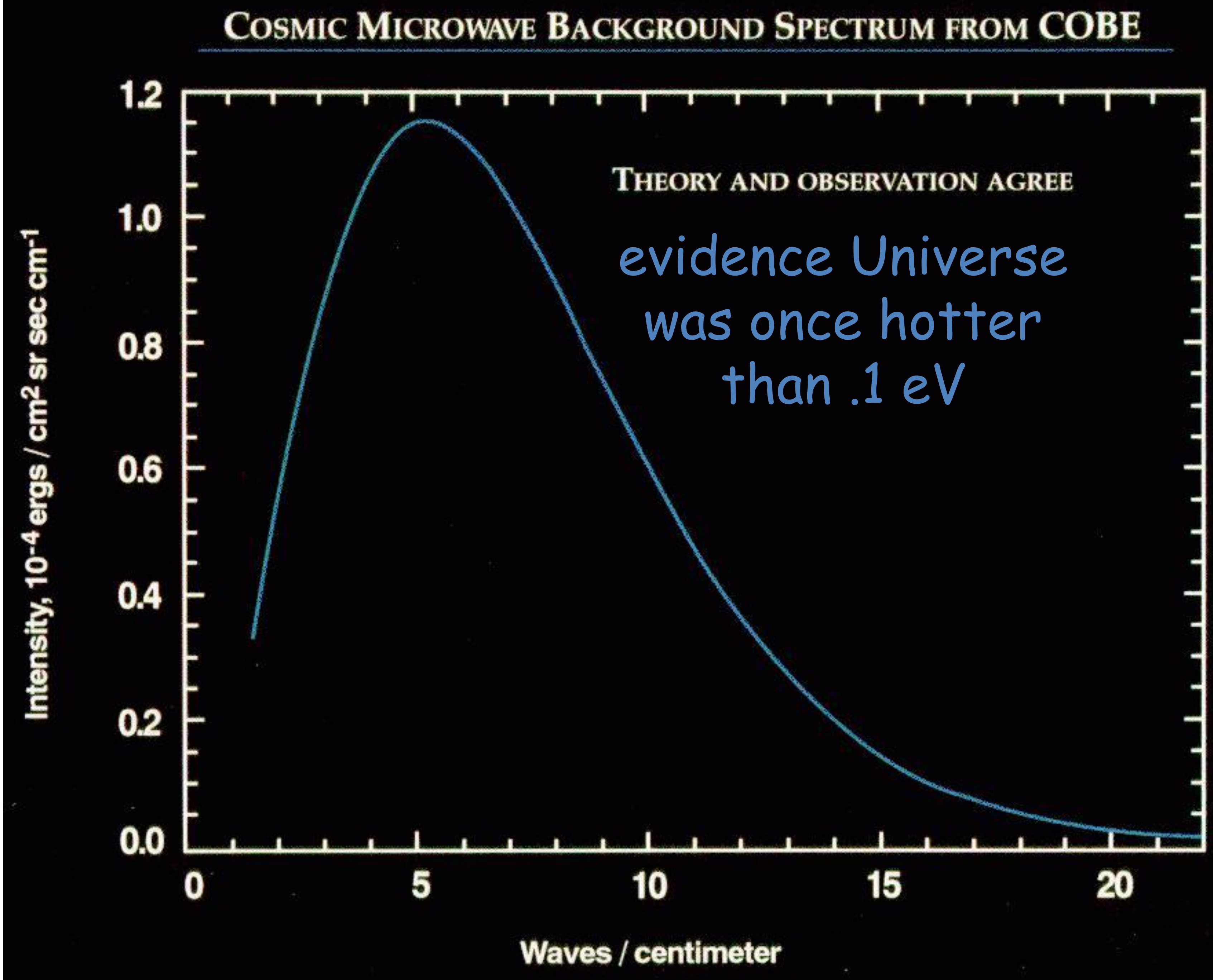
$$T=2.725K$$

Thermal History

- For photons, $\rho \propto a^{-4}$ (from Friedmann equations) and $\rho \propto T^4$ (from Stefan-Boltzmann law), thus $T \propto a^{-1}$
- Energy of photons: $E = hc/\lambda$ and $\lambda \propto a$, so $E \propto a^{-1}$
- Planck black-body spectrum: $dn \propto \frac{E^2 dE}{\exp(E/T)-1}$
- Since $E \propto a^{-1}$ and $T \propto a^{-1}$, $E/T = E'/T'$; also, $E^2 dE \propto a^{-3}$ and $dn \propto a^{-3}$

$$\Rightarrow dn' \propto \frac{E'^2 dE'}{\exp(E'/T') - 1}$$

Photons retain black-body spectrum as Universe expands



evidence Universe
was once hotter
than .1 eV

- Note that the Planck black-body spectrum remains even after photons drop out of thermal equilibrium (decoupling at $z \sim 1000$)

- At high temperatures, elementary particles are relativistic, thus

$$\rho = \frac{\pi^2}{30} g_{tot} T^4$$

where g_{tot} is the sum of effective degrees of freedom.

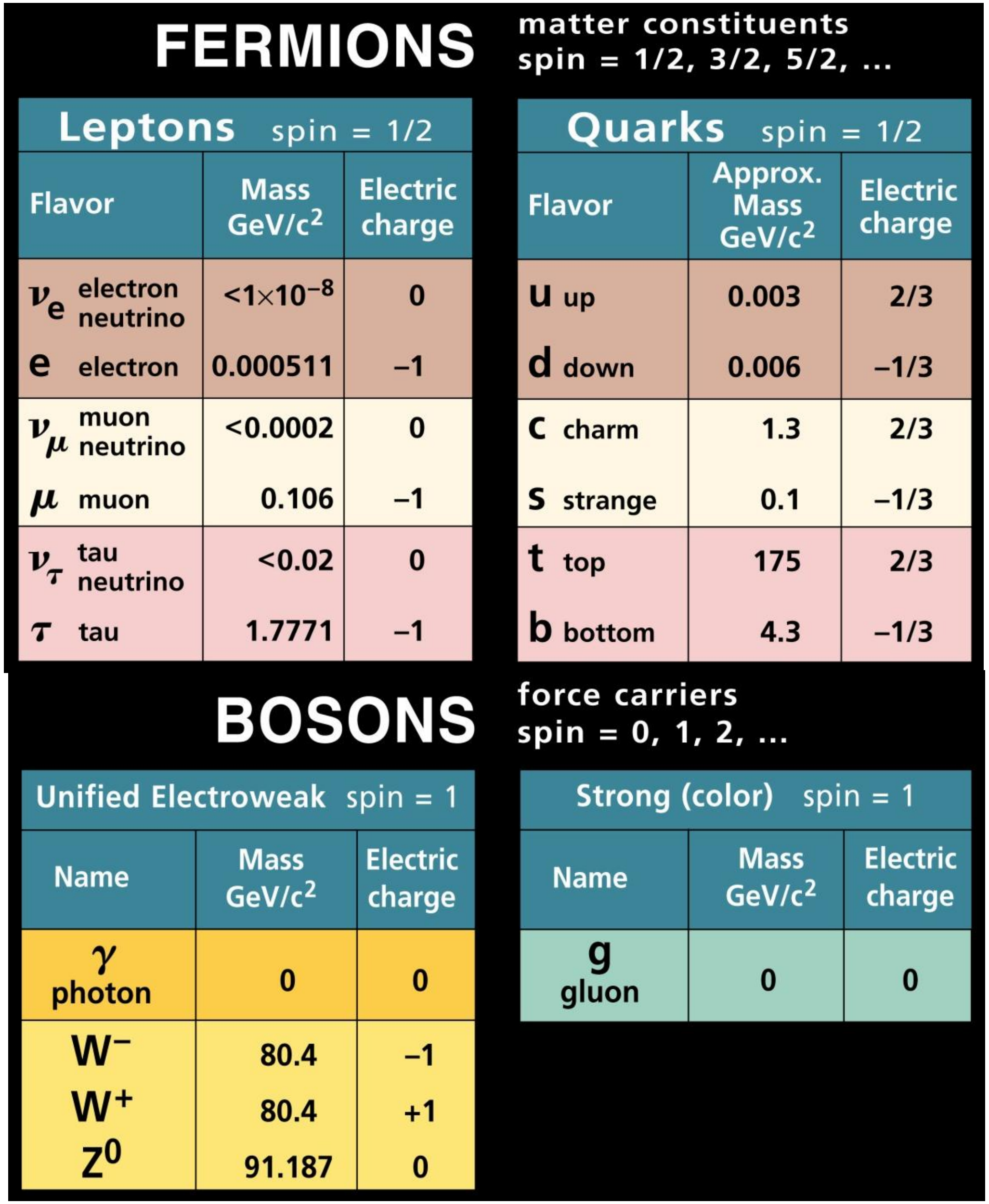
- In radiation-dominated era, also have

$$\rho = \frac{3H^2}{8\pi G} = \frac{3}{32\pi G t^2} \; (a \propto t^{\frac{1}{2}} \rightarrow H = \dot{a}/a = 1/2t)$$

- Thus temperature can be used as a cosmic clock:

$$\frac{t}{1\, sec} \approx g_{tot}^{-\frac{1}{2}} \left(\frac{1\, MeV}{T}\right)^2$$

Time	Temperature	Process
$\sim 10^{-38} s$	$\sim 10^{12} GeV$	GUT scale: electroweak decouples from strong force
$\sim 10^{-8} s$	$\sim 10 - 100\ GeV$	WIMPs decouple (if they exist and are the dark matter)
$\sim 10^{-5} s$	$\sim 100 - 300\ MeV$	Quark-hadron phase transition: neutrons and protons created from quarks and gluons
$\sim sec - min$	$\sim 10 - 0.1\ MeV$	Nucleosynthesis: protons and neutrons join to create atomic nuclei
$\sim 10^{12} s$	$\sim 3\ eV, \sim 10^4\ K$	Matter-radiation equality
$\sim 400{,}000\ yr$	$\sim eV, \sim 3000\ K$	Recombination: electrons and protons form to create Hydrogen atoms, and CMB photons decouple



FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

BOSONS

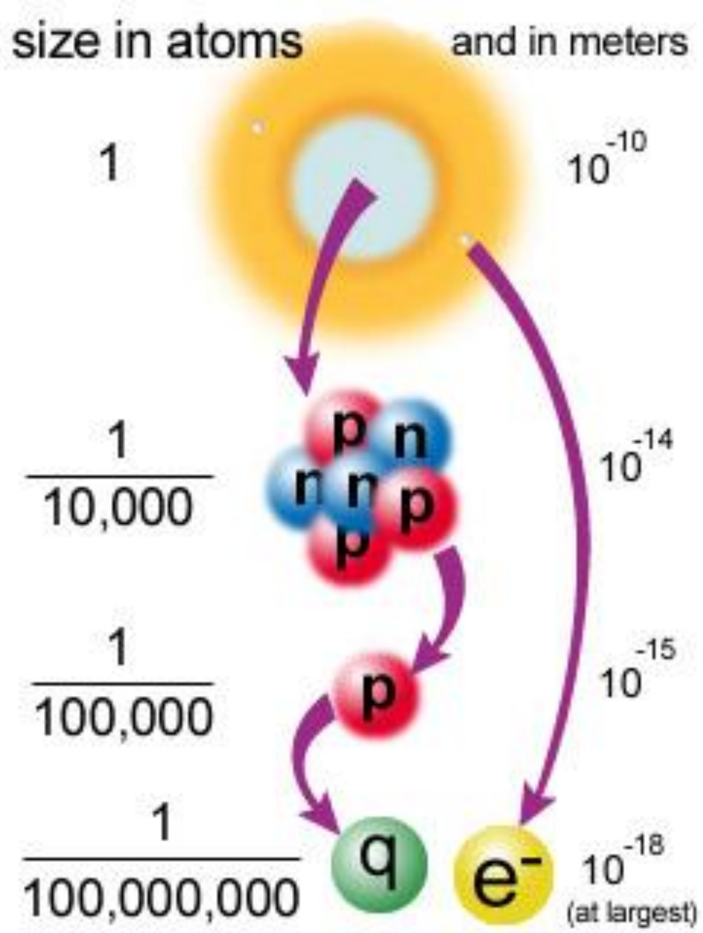
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.4	–1
W$^+$	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0



http://particleadventure.org/particleadventure/index.html

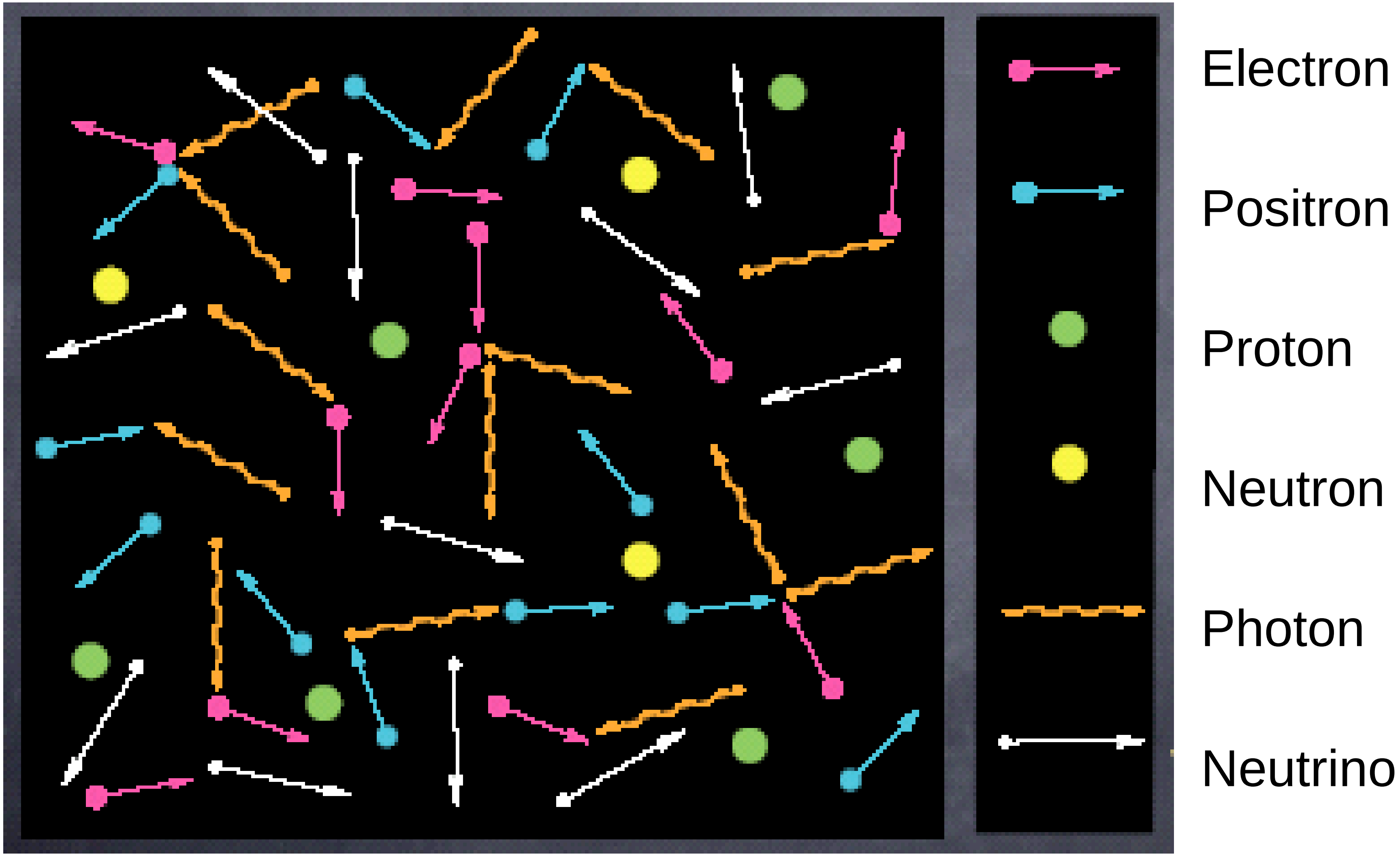
- Electroweak phase transition
  - quarks and leptons (e.g. electron, neutrino)

- QCD (quantum chromo-dynamics) phase transition
  - quark-gluon plasma → confinement of quark

proton

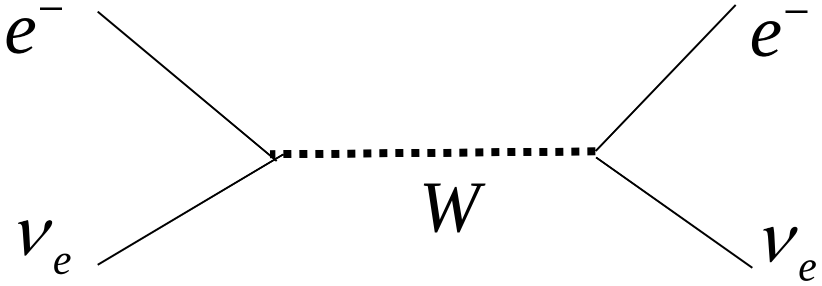
neutron

We assume there is an asymmetry between matter and anti-matter.



(figure from Kazuya)

- Neutrino decoupling
  - Neutrino interacts via weak interaction:



  - Reaction rate:

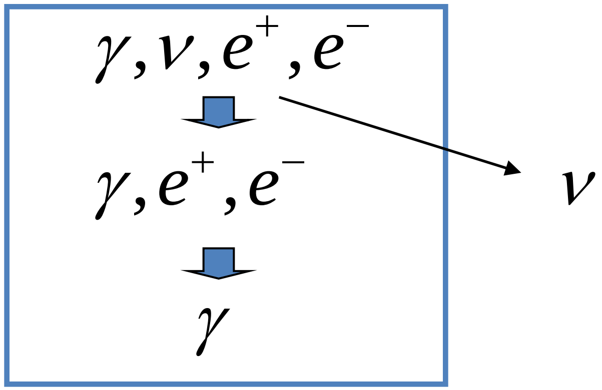
$$\Gamma \approx G_F{}^2 T^5, \;\; G_F \approx 1.17\times 10^{-5}\,\mathrm{GeV}^{-2} \approx m_W{}^{-2}$$

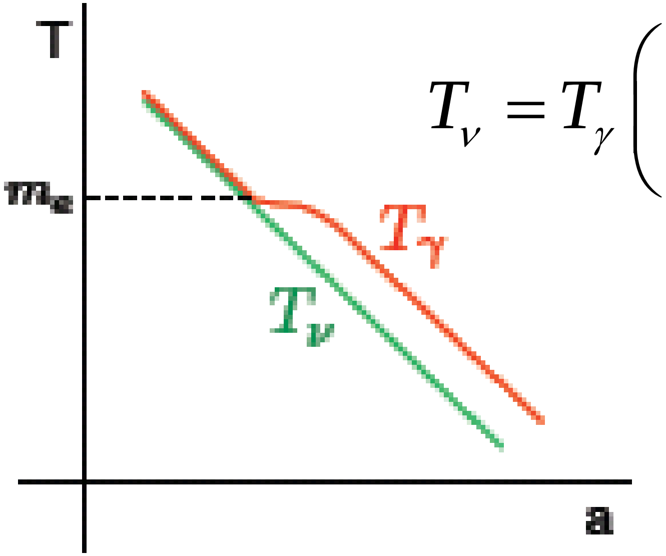
  - Decoupling occurs when:

$$\frac{\Gamma}{H} \approx \left(\frac{T}{1.4\mathrm{MeV}}\right)^3 < 1$$

- Electron-positron annihilation: $e^+ + e^- \to \gamma$
  - When the temperature drops below $T < m_e$, photon cannot produce e^+, e^-
  - Tiny amounts of electron remain (Lepton asymmetry)

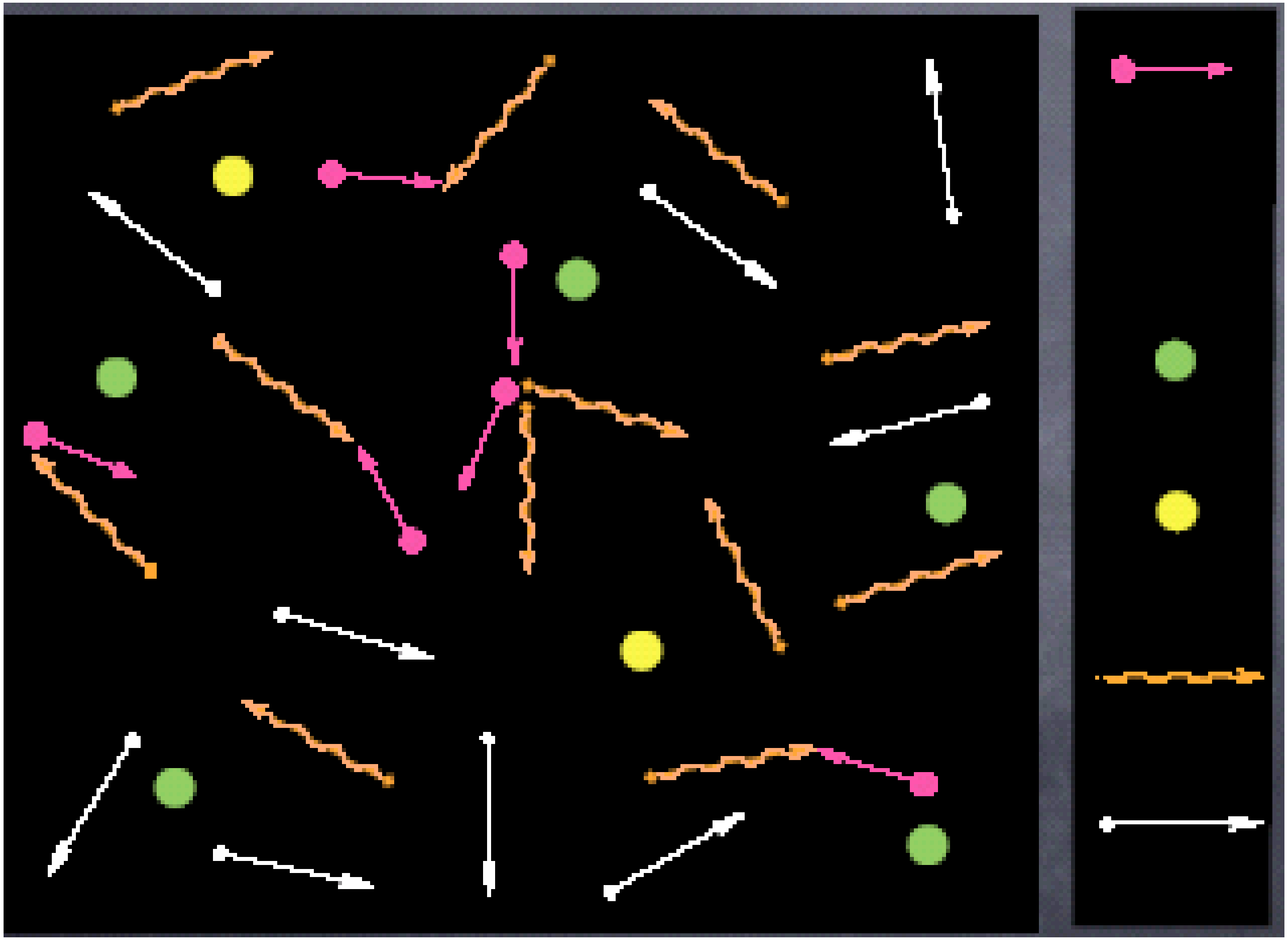
Thermal Bath:





$$T_\nu = T_\gamma \left(\frac{4}{11}\right)^{\frac{1}{3}}$$

  - Photon temperature changes due to $e^+ + e^- \to \gamma$



Electron

Proton

Neutron

Photon

Neutrino

Nucleosynthesis

- Light stable nuclei synthesized from free protons and neutrons
  - Deuterium: p + n → D $E = 2.2\ MeV$
  - Helium-3: D + p → ^{3}He $E = 7.7\ MeV$
  - Helium-4: D + D → ^{4}He $E = 28\ MeV$

- Before nucleosynthesis, non-relativistic baryons kept in statistical equilibrium by weak interactions:

$$\text{scattering}\ \left\{\begin{array}{l} n + e^{+} \rightarrow p + \bar{\nu}_e, \\ p + e^{-} \rightarrow n + \nu_e, \end{array}\right.$$

$$\text{decay}\qquad n \rightarrow p + e^{-} + \bar{\nu}_e$$

- The relative number of neutrons to protons is given by the Boltzmann factor,

$$\frac{n_n}{n_p} \cong \exp\left(-\frac{Q}{kT}\right)$$

where $Q = (m_n - m_p)c^2 = 1.3\ MeV$

- At high temperatures $(kT \gg Q, m_e c^2), \Gamma \propto G_F^2 T^5$
  - Protons freeze out at $T \cong 0.8\ MeV$, after which only neutron decay important
  - This gives $n_n/n_p \cong 1/6$

- Because of large photon to baryon ratio, stable nuclei do not form until $T < 0.1\ MeV$ *(deuterium)*
  - By then, neutron decay gives $n_n/n_p \cong 1/7$

- Deuterium production

$$n + p \rightarrow D + \gamma$$

  - Binding energy $Q_D = 2.2\ MeV$
  - If number density of photons with $E > Q_D$ decreases, deuterium production starts with $T \ll Q_D$

- Helium production

$$\begin{aligned} D + D &\rightarrow {}^{3}He + n, \\ {}^{3}He + n &\rightarrow {}^{3}H + p, \\ {}^{3}H + D &\rightarrow {}^{4}He + n \end{aligned}$$

  - Note that the tritium isotope of Hydrogen has 2n + p

- Binding energy of Helium-4 is 28 MeV, so essentially all neutrons become processed into Helium-4
  - Can estimate primordial abundance ratio:

$$Y = \frac{n_{^4He}}{n_B} = \frac{4\left(\frac{n_n}{2}\right)}{n_n + n_p} = \frac{2\left(\frac{n_n}{n_p}\right)}{1 + \frac{n_n}{n_p}}$$

- Recall that $\frac{n_n}{n_p} \cong \exp\left(-\frac{Q}{T_F}\right) \approx \frac{1}{7}$

$$\Longrightarrow Y \cong 0.25$$

- Observationally, $0.22 < Y < 0.25$

- Heavier light elements are not formed
  - No stable nuclei with atomic number A = 5, 8
  - Tiny amounts of Lithium are produced

$$^{4}He + {}^{3}H \rightarrow {}^{7}Li + \gamma,$$

$$^{4}He + {}^{3}He \rightarrow {}^{7}Be + \gamma,$$

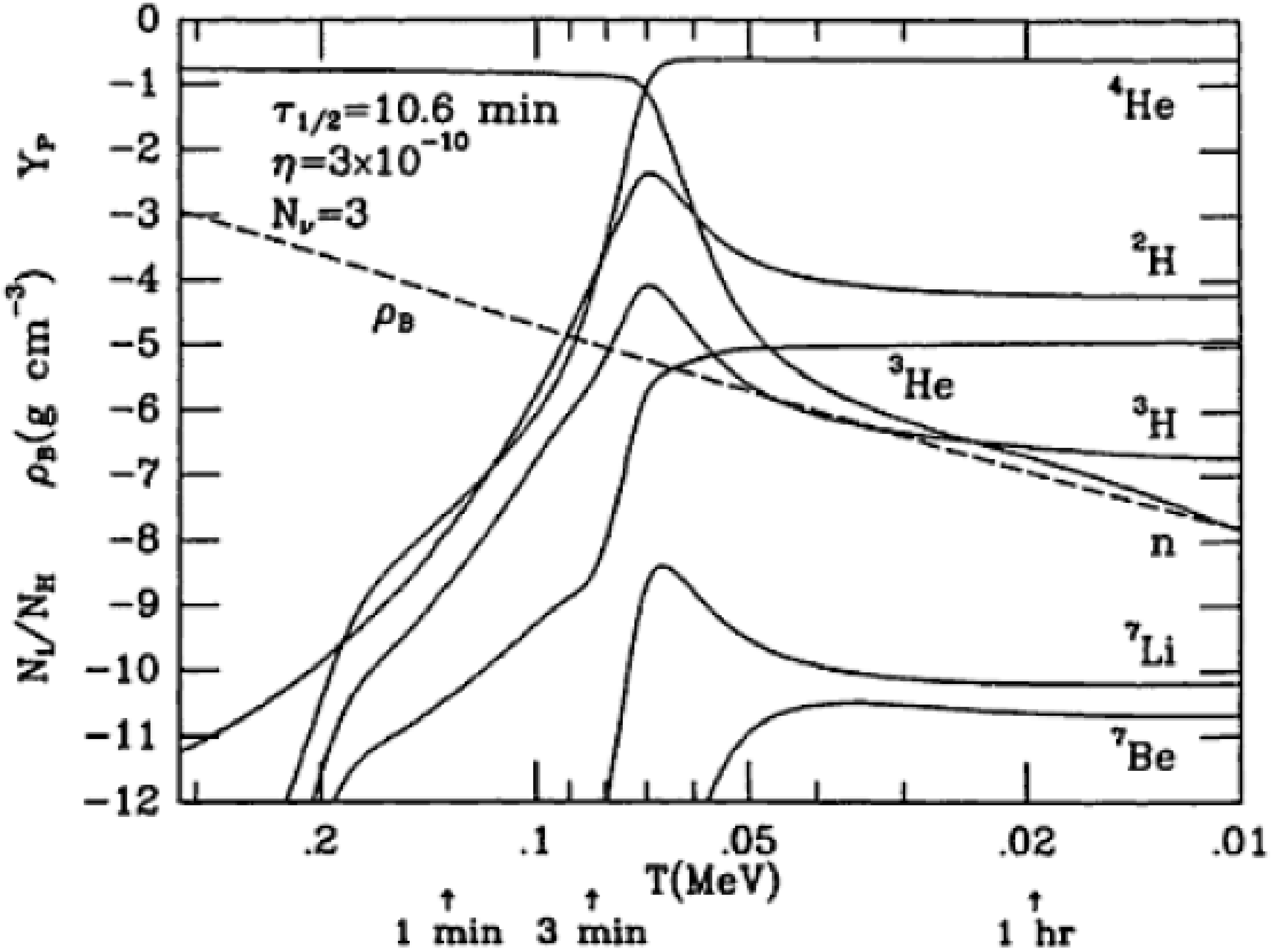
$$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_{e}$$

- Density of Helium is too small for “triple-α” process, ^{4}He + ^{4}He + ^{4}He → ^{12}C, which occurs in stars

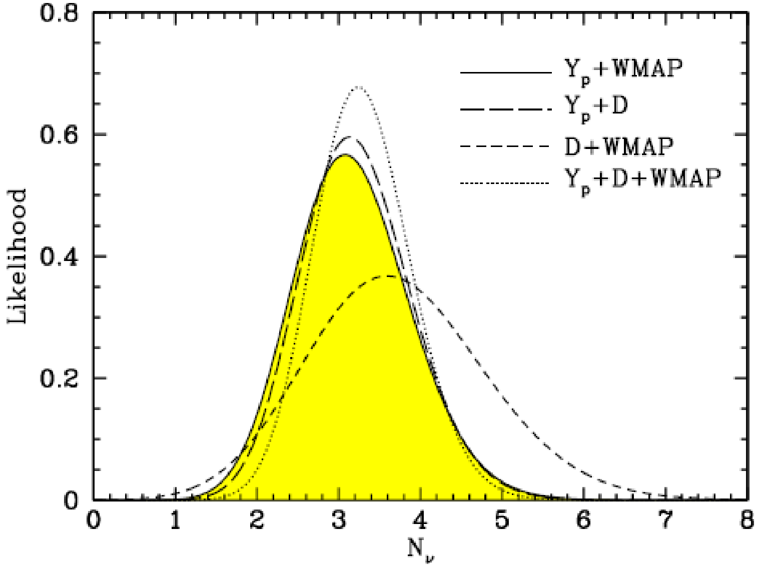
- Abundance of light elements depends only on baryon to photon fraction

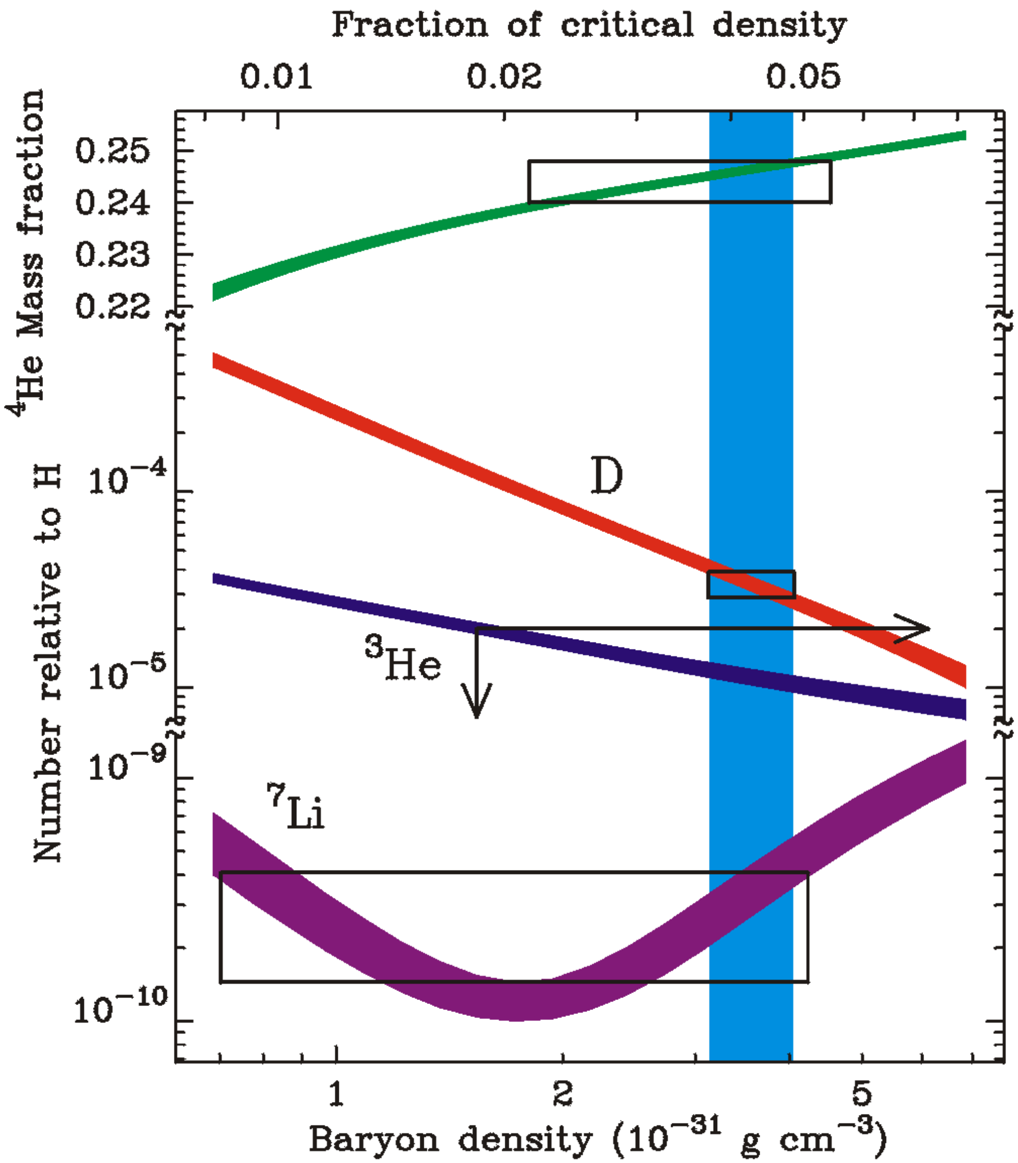
$$\eta = \frac{n_B}{n_\gamma} = \frac{n_n + n_p}{n_\gamma} = 2.68 \times 10^{-8}\Omega_B h^2$$

Evolution of light element abundances in the early universe



- Nucleosynthesis constrains e.g. number of neutrino degrees of freedom, g_*
  - If g_* were larger, expansion rate $H^2 \propto g_*^{\frac{1}{2}} T^2$ would be larger at same temperature
    - ➢ Neutron-proton conversion would freeze out earlier at higher T thus higher n/p
    - ➢ Larger abundance of Helium-4
- Conservatively, $g_* \leq 4$

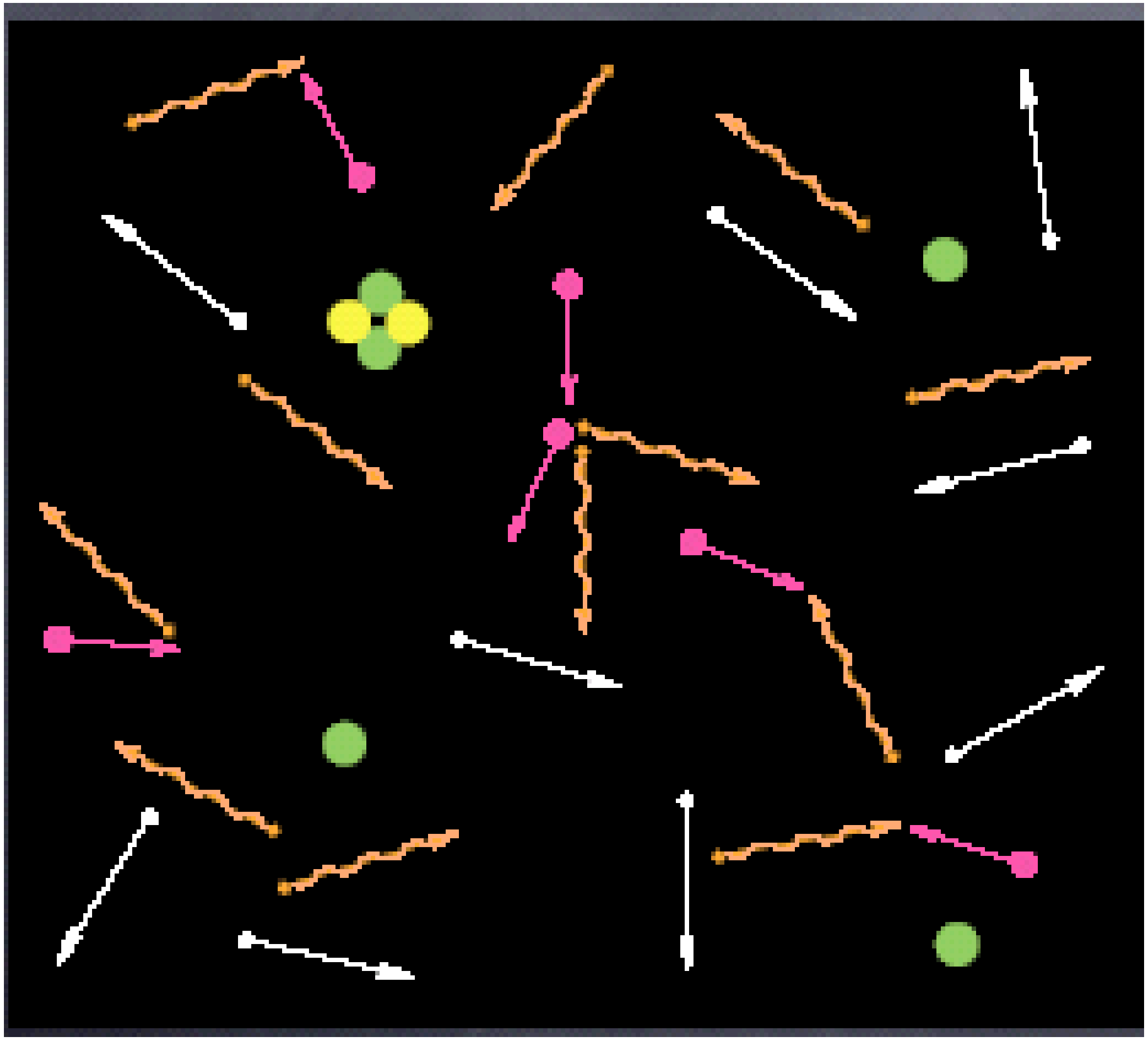




Observed abundances of Helium-3 and Deuterium constrain baryon/photon ratio:

$$\eta \equiv (5.8 - 7.5) \times 10^{-10}$$

The CMB measurements provide independent information for η

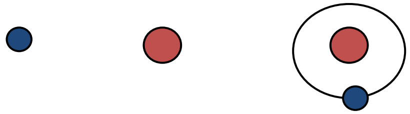


Now we have free-streaming neutrinos, interacting electrons and photons, and light nuclei

Cosmic Microwave Background

- The process of forming neutral hydrogen as a proton captures an electron is called recombination

$$e^{-} + p \rightarrow H + \gamma$$



- As long as the reaction is in thermal equilibrium:

$$\frac{n_p n_e}{n_H} \approx \left(\frac{m_e T}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{13.6\text{eV}}{\text{T}}\right)$$

$$\boxed{\begin{aligned}&(m_p + m_e) - m_H\\ &= 13.6\text{eV}\end{aligned}}$$

- Define the ionization fraction:

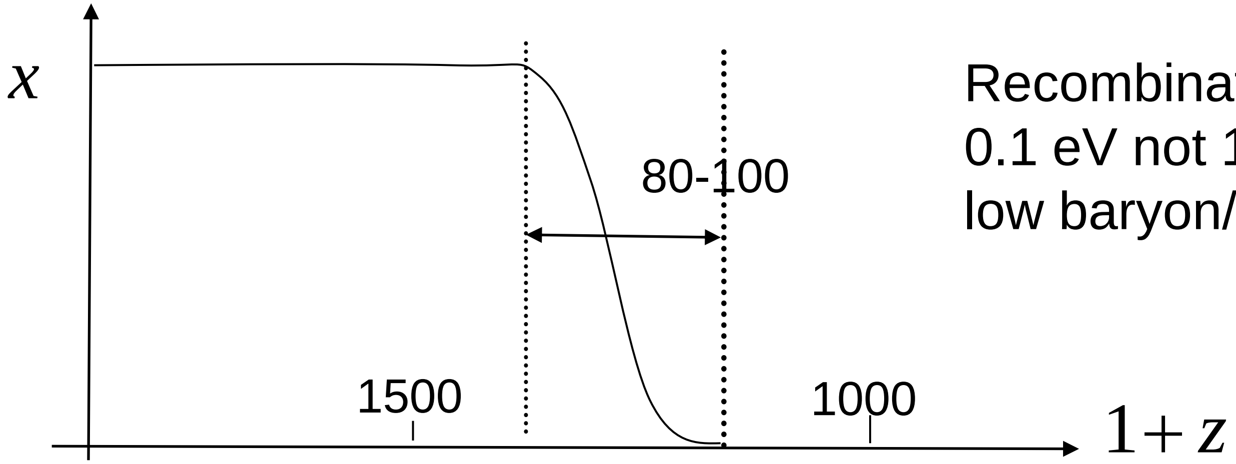
$$x \equiv \frac{n_e}{n_B} = \frac{n_e}{n_p + n_H} = \frac{1}{1 + \frac{n_H}{n_e}}, \qquad (n_p = n_e)$$

- The ionization fraction evolves according to the Saha equation,

$$\frac{n_p n_e}{n_H n_B} = \frac{1}{n_B}\left(\frac{m_e T}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{13.6\mathrm{eV}}{\mathrm{T}}\right) = \frac{x^2}{1-x}$$

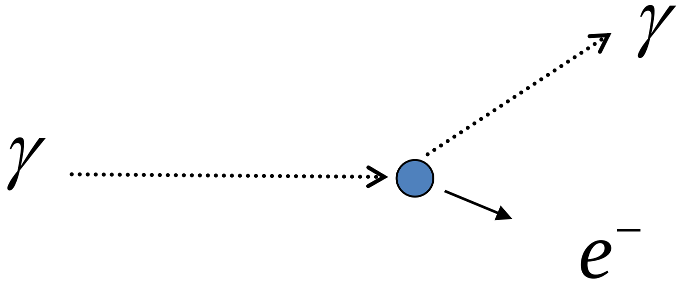
- Using the baryon to photon ratio, this can be rewritten as

$$\frac{x^2}{1-x} \approx \eta^{-1}\left(\frac{T}{m_e}\right)^{-\frac{3}{2}} \exp\left(-\frac{13.6\,\mathrm{eV}}{T}\right)$$



Recombination occurs at 0.1 eV not 13.6 eV due to low baryon/photon ratio

- Decoupling
  - As the ionization fraction → 0, Thomson scattering becomes ineffective

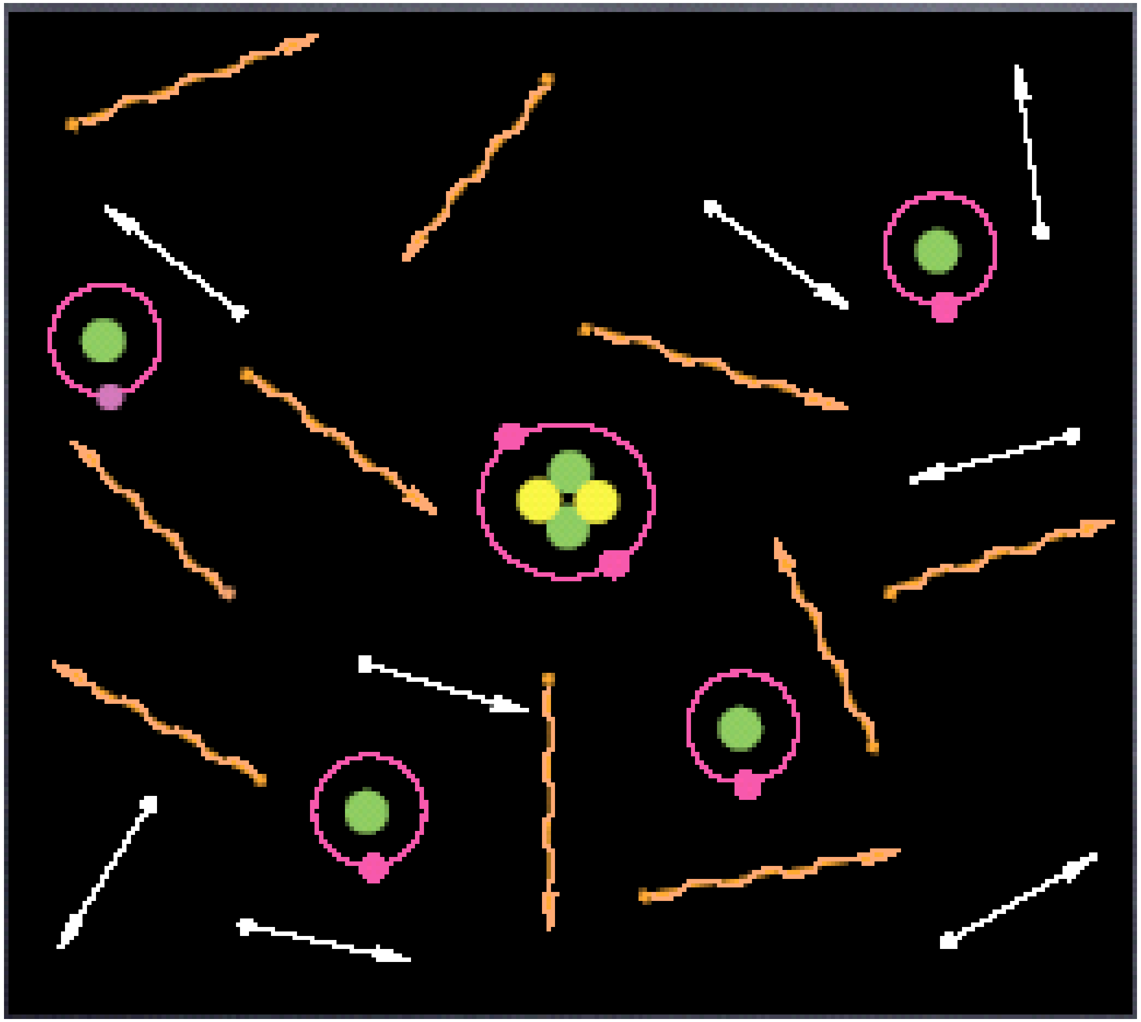


- The scattering rate is

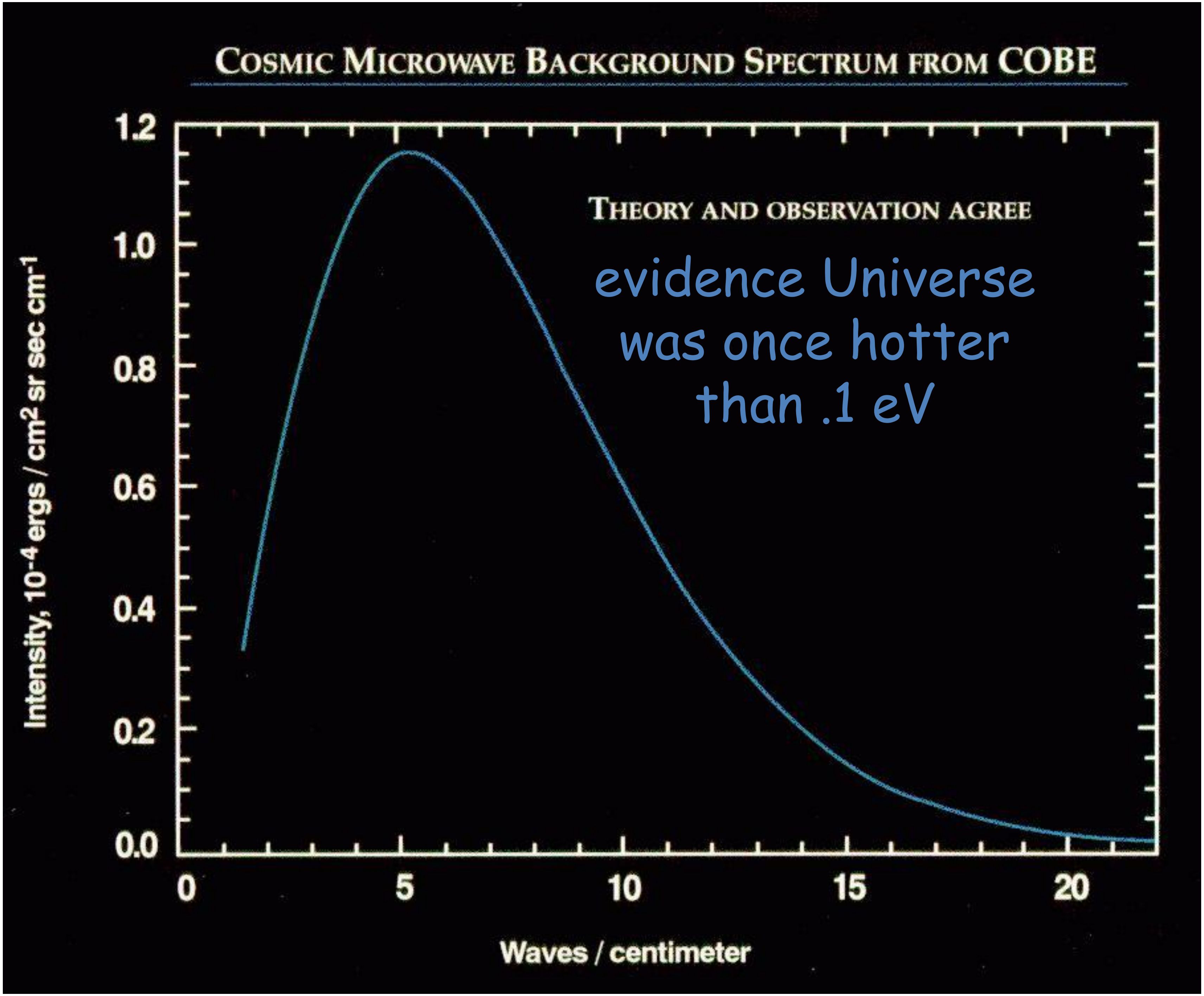
$$n_e \sigma_T = x_e n_B \sigma_T \propto x_e \Omega_B h^2 a^{-3}$$

- Photons decouple when $x_e \sim 10^{-2}$
  - Decoupling occurs during the process of recombination

- This "last scattering surface" occurred at $z \sim 1000, T \sim 3000\ K$



Now we have free-streaming neutrinos and photons, and atomic hydrogen and helium

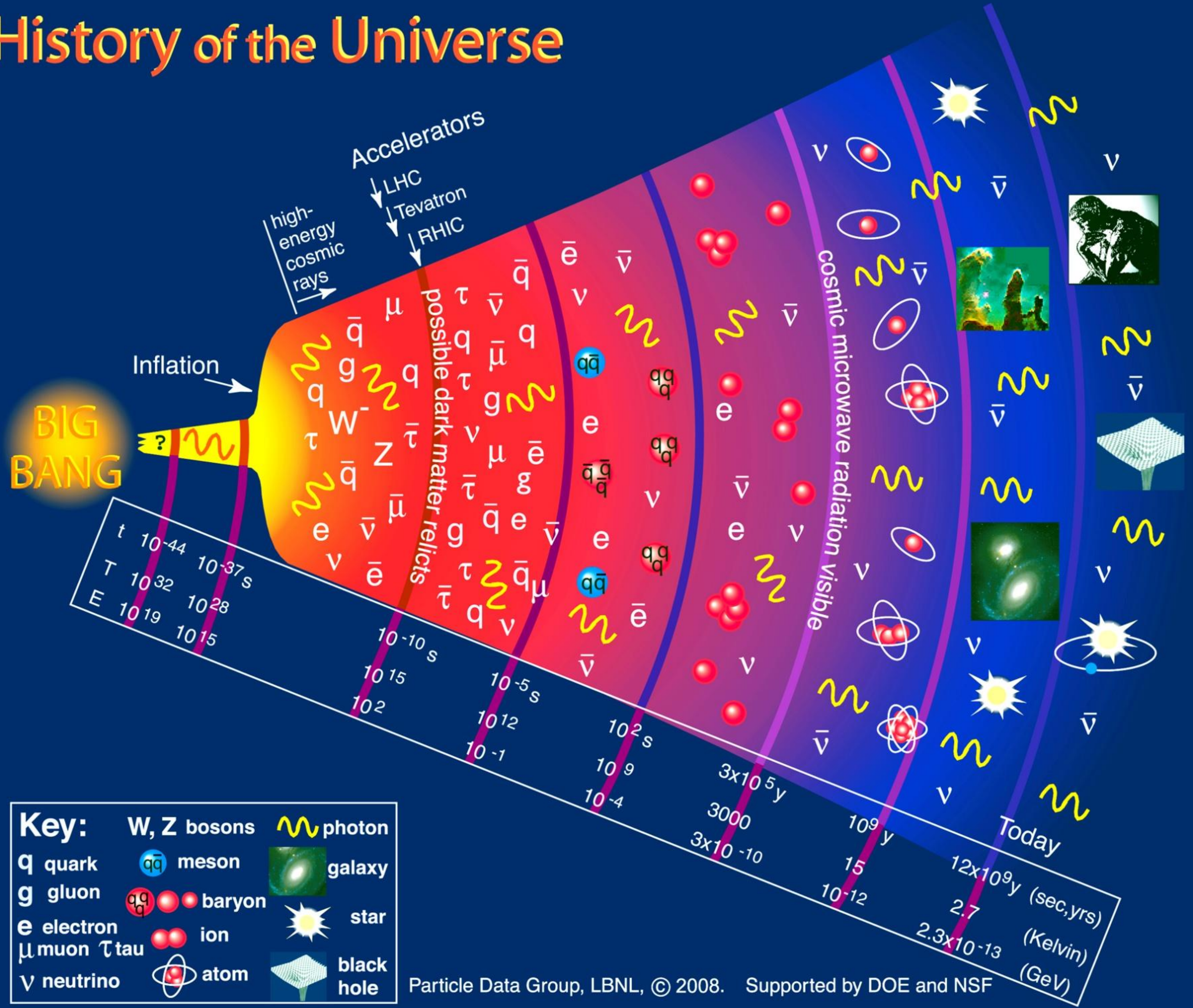


COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE

THEORY AND OBSERVATION AGREE

evidence Universe
was once hotter
than .1 eV

History of the Universe



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