Problems with the Standard Model of Cosmology

Initial Conditions
Dark Matter
Dark Energy
The Standard Model of Cosmology

1. The hot big bang + expansion of the Universe
   - explains nucleosynthesis and the CMB

2. + dark matter and dark energy
   - explain the growth of structures and distances to bright objects
The LCDM Model

- The standard cosmological model, LCDM, explains observations consistently in a simple framework—but we do not understand its components
  - Need *inflation* or some other theory to explain flatness of geometry and smoothness of CMB
  - We haven’t detected *dark matter* and don’t know what it is (it’s outside the standard model of particle physics)
  - We don’t know what *dark energy* is or why the value of the cosmological constant is 120 orders of magnitude off
Initial Conditions Problems

- **Flatness** problem: Why is $\Omega_0 = 1$?
  - WMAP: $\Omega_0 = 1.003 + 0.013 - 0.017$

- **Horizon** problem: Why is CMB so homogeneous?
  - $\frac{\Delta T}{T} \sim 10^{-5}$

- **Density perturbations**: What is the origin of $\frac{\Delta T}{T}$?
The Flatness Problem

- **Friedmann equation:**
  \[ H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho \]
  \[ \Rightarrow |\Omega_{tot}(t) - 1| = \frac{|K|}{a^2 H^2} \]

  - During radiation domination: \( |\Omega_{tot}(t) - 1| \propto t \)
  - During matter domination: \( |\Omega_{tot}(t) - 1| \propto t^2 \)

- Thus flat geometry is **unstable** solution
  - Requires extremely small \( |\Omega_{tot}(t) - 1| \) in early universe
The Horizon Problem

- Due to the finite age of the Universe, the size of causally connected regions is also finite, known as the horizon.
  - Co-moving horizon: $c \tau$, where $\tau$ is the conformal time.
  - Physical horizon: $d_H = a(t) \int_0^t \frac{c dt'}{a(t')}$

- If $a \propto t^n$,
  \[ d_H = \frac{ct}{1-n} = \frac{n}{1-n} \left( \frac{c}{H} \right) \]
  - Radiation dominated: $a \propto t^{\frac{1}{2}} \rightarrow d_H = 2ct = c/H$
  - Matter dominated: $a \propto t^{\frac{2}{3}} \rightarrow d_H = 3ct = 2c/H$
• Today, the physical horizon is $\sim 6$ Gpc, but at the time of the last scattering surface it was much smaller.

• Causally connected region of CMB subtends an angle

$$\theta \equiv (1 + z_{\text{LSS}}) \left( \frac{t_{\text{LSS}}}{t_0} \right) \sim 2^\circ$$

Why do causally disconnected patches of CMB have the same temperature to $10^{-5}$?
In terms of the conformal time $\tau = \int dt/a$, if $a \propto t^n$,

$$\tau = \frac{t^{1-n}}{1-n}$$

\[ \text{(t = 0 if } n < 1) \]
Density Perturbations

1. The hot big bang model does not account for origin of small initial density perturbations.

2. There are CMB perturbations whose wavelengths are larger than the horizon at last scattering (thus acausal).
Inflation

- From the acceleration equation (neglecting \( \Lambda \)),
  \[
  \ddot{a} = -\frac{4\pi G}{3} (\rho + 3p)
  \]

  \( \ddot{a} > 0 \) requires \( w < -1/3 \), where \( p = w\rho \)

- If \( w = -1, a \propto \exp(\dot{H}t) \) and
  \[
  d_H = \frac{1}{H} (e^{\dot{H}t} - 1)
  \]

Some extra scalar field required to start (and stop) inflation
The Horizon Problem:

During matter and radiation domination, $\dot{a} < 0$, so
\[
\frac{d}{dt} \left[ \frac{1}{\dot{a}} \right] = \frac{d}{dt} \left[ \frac{1}{aH} \right] > 0
\]

Thus the Hubble radius, $c/H$, increases faster than the scale factor
- Objects at Hubble radius have recession velocity = $c$

During acceleration, $\ddot{a} > 0$, instead structures once smaller than the Hubble radius become larger and “leave the horizon”
- No longer in causal contact; solves horizon problem
Expansion

Structures enter the Hubble radius “horizon”

Inflation

Structures leave the “horizon”
• Acceleration also requires $n > 1$, where $a \propto t^n$

$$\rightarrow \tau = \int \frac{dt}{a} \propto \frac{t^{1-n}}{1+n} \rightarrow \infty$$
• How long should inflation last?
  – Physical scale of particle horizon at end of inflation must be larger than Hubble distance now:

\[
\frac{1}{a_f H_{\text{inf}}} e^{H_{\text{inf}}(t_f-t_i)} > \frac{1}{a_0 H_0}
\]

• E-folding number: universe expands by factor of \(e^N\)

\[
N = H_{\text{inf}}(t_f - t_i) = \ln\left(\frac{a_f}{a_i}\right) \approx 60
\]

  – Can be very quick, e.g. \(t_i = 10^{-36}\) s to \(t_f = 10^{-34}\) s
• The Flatness Problem:

\[ |\Omega_{tot}(t) - 1| = \frac{|K|}{a^2H^2} \]

- During radiation domination: \( a \propto t^{\frac{1}{2}} \)
  \[ \rightarrow |\Omega_{tot}(t) - 1| \propto t \]

- During inflation: \( a \propto e^{Ht} \)
  \[ \rightarrow |\Omega_{tot}(t) - 1| \propto e^{-2Ht} \]

• If number of e-foldings \( N > 60 \), the geometry becomes incredibly flat during inflation
• **Density perturbations:**

1. Inflation gives the natural primordial perturbations: quantum fluctuations
2. It is natural to have super-horizon perturbations
Initial Singularity:

- $t = 0$ is curvature singularity: $R_{\mu\nu\rho\sigma} \to \infty$

Energy density exceeds Planck energy,

$$E_{Pl} \equiv \sqrt{\frac{\hbar c^5}{G}} = 1.22 \times 10^{19} \text{GeV}$$

- Need quantum theory of gravity

- But Planck time is much earlier than inflation,

$$t_{Pl} \equiv \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44} \text{sec}$$

- Inflation independent of singularity
Dark Matter

- WIMP = weakly interacting massive particle
  - SUSY predicts as lightest super-symmetric particle
  - But SUSY disfavored by LHC?

- “WIMP miracle” – correct abundance requires cross-section which is roughly what’s expected for weak scale particle ~ 100 GeV

100 GeV predicts $\Omega_{m} \sim 0.3$
• **Direct detection:**

\[ \beta \approx 10^{-3} \]

\[ T_{\text{max}} \approx 2 M \alpha c^2 \beta^2 \]

- CDMS
- CoGeNT
Confusing results...

detections

exclusions
• **Indirect detection**: Positron excess a hint of DM annihilation or local pulsars?
Galaxy Formation

- **Cusp-Core problem**: DM halo profile fits cuspy NFW in simulations but rotation curves of galaxies have a core

- Likely to be resolved by galactic physics
Galaxy Formation

• **Missing Satellites problem**: Many more satellites in CDM simulations than observed
  
  – Warm dark matter would suppress formation of satellites
  – Satellites may be too faint or contain only dark matter
Galaxy Formation

• “Too big to fail” problem: simulations predict many more massive subhalos than could be hosts of the Milky Way’s brightest satellites

  – They aren’t too faint to be observed, thus “too big to fail”

Dark Energy

- **Cosmological constant problem**: quantum field theory predicts huge value for vacuum energy, but

\[ \Lambda \approx H_0^2 = (10^{-42} \text{ GeV})^2 = 10^{-120} M_{pl}^2 \]

- **Coincidence problem**: why is the era of accelerated expansion happening *now*?
Models of Acceleration

- The two main types of attempts to model acceleration depend on which side of Einstein’s equation you change:

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R[+?] = 8\pi G T_{\mu\nu}[\Lambda g_{\mu\nu}] \]

- Dark energy models add a new component such as the vacuum energy
- Modified gravity models tweak the left hand side

- More complicated models exist, or you can ignore the problem completely via the *anthropic landscape*
Models of Acceleration

- **Quintessence**: new degree of freedom, scalar field $\phi$ with potential $V(\phi)$, makes vacuum energy effectively dynamical.

\[ w_q = \frac{\dot{\phi}^2}{2} - V(\phi) \]

- Many models have **tracker solution**: density tracks radiation until “turns on” at matter-radiation equality.

Frieman, et al. 2008, ARAA, 46, 385
Models of Acceleration

- **Modified gravity**: tweak GR at cosmological scales, where it has not been tested
  - Many (not all) require screening mechanism to satisfy local constraints

- **Ex: $f(R)$ gravity** introduces function of Ricci scalar in GR action
  - Chameleon screening increases mass of scalar field in high density environments

Jain & Khoury 2010
The Anthropic Landscape

• The anthropic principle is an observation that the Universe we observe is consistent with the ability to produce us as observers.

• Some take that as a *prediction*: various fundamental constants, etc. including the value of $\Lambda$ *must* be such as to allow intelligent life.

• String theory has $10^{500}$ solutions to their equations, so there is a *multiverse* and we live in the universe capable of producing us.

• These arguments have *serious philosophical issues*.